

Middle Mile Transportation Network and Optimal Replenishment for a Supply Chain
Solutions Company

by

I-Chen Hsieh

Bachelor of Arts, Economics, National Tsing Hua University, 2021

and

Bharti Yadav

Post Graduate Diploma in Industrial Engineering, Indian Institute of Management Mumbai, 2016

SUBMITTED TO THE PROGRAM IN SUPPLY CHAIN MANAGEMENT
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF APPLIED SCIENCE IN SUPPLY CHAIN MANAGEMENT
AT THE
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

May 2025

© 2025 I-Chen Hsieh and Bharti Yadav. All rights reserved.

The authors hereby grant to MIT permission to reproduce and to distribute publicly paper and electronic copies of this capstone document in whole or in part in any medium now known or hereafter created.

Signature of Author: I-Chen Hsieh
Department of Supply Chain Management
May 9, 2025

Signature of Author: Bharti Yadav
Department of Supply Chain Management
May 9, 2025

Certified by: Milena Janjevic
Milena Janjevic
Research Scientist at the MIT Center for Transportation & Logistics
Capstone Advisor

Accepted by: _____
Prof. Yossi Sheffi
Director, Center for Transportation and Logistics
Elisha Gray II Professor of Engineering Systems
Professor, Civil and Environmental Engineering

Middle Mile Transportation Network and Optimal Replenishment for a Supply Chain Solutions Company

by

I-Chen Hsieh

and

Bharti Yadav

Submitted to the Program in Supply Chain Management
on May 9, 2025 in Partial Fulfillment of the
Requirements for the Degree of Master of Applied Science in Supply Chain Management

ABSTRACT

The sponsor company faced challenges in optimizing middle-mile replenishment frequency due to the inevitable trade-off between transportation and inventory carrying costs. To address this challenge, we conducted a comprehensive analysis comparing transportation and inventory costs across multiple replenishment frequency scenarios. Additionally, we implemented a high-performance vehicle routing problem (VRP) solver to redesign the middle mile transportation network, reducing total distance traveled by over 2 million miles a year and net working capital (NWC) by over 80 trucks every replenishment cycle. Our deliverables enabled the company to improve operational efficiency and reduce costs through optimized routing and replenishment strategies. As they expand their business, the company can continue to generate cost-effective, data-driven decisions by inputting new data into the flexible model we developed.

Capstone Advisor: Milena Janjevic

Title: Research Scientist, MIT Center for Transportation & Logistics

ACKNOWLEDGMENTS

Joint Acknowledgements:

We would like to express our sincere gratitude to our advisor, Dr. Milena Janjevic, for invaluable guidance and unwavering support throughout our capstone project. This 9-month journey would have been far more challenging without your mentorship. Your expertise in network design, encouragement during difficult moments, and firm belief in us made all the difference.

We would also like to thank our sponsor company for timely responses and ongoing collaboration. The successful completion of this project would not have been possible without your support and consistent communication with us.

We appreciate the support of our writing coach, Pamela Siska. Your attention to detail, expertise in refining and structuring our writing, and heartfelt encouragement were essential to us throughout the process.

Lastly, we extend our special thanks to our Network Design TA, Austin Saragih, for introducing us to PyVRP. Your guidance in helping us learn and implement the latest VRP solver was instrumental to our project's success. We truly could not have done it without your generous support.

I-Chen (Genny) Hsieh Acknowledgements:

My deepest gratitude goes to my parents, Yen-Chi Kuo and Wen-Chiung Sheih. Your faith in me has been a quiet constant and your love has shaped every step of my journey. Your unconditional support, enduring encouragement, and lifelong investment in my education have made everything possible. I am profoundly fortunate and forever grateful.

I would also like to express my heartfelt appreciation to my friends, including those back home and the ones I've met here at MIT. You brought light into my life during the darkest and most challenging moments of this journey. Your presence, laughter, and emotional support carried me through. I truly couldn't have done it without you.

Finally, I would like to thank my boyfriend. Thank you for supporting my decision to pursue a master's degree in the U.S., even though it meant being in a long-distance relationship. Your patience in teaching me how to debug, your calm presence during my anxious moments, your belief in me when my confidence faltered, and the way you caught me whenever I felt like falling meant the world to me.

Bharti Yadav Acknowledgements:

I am profoundly grateful to my parents, Krishna and Kailash Yadav. My mother's quiet strength and resilience have been my anchor through every high and low. My father's dreams and values continue to guide and inspire me each day. Their unwavering belief in me gave me the courage to believe in myself, and I carry their love and lessons with me always.

I would also like to thank Professor Sean Willems. Thank you for making inventory management something I genuinely enjoy. Your passion for the subject and the way you taught it helped me connect with it deeply, and it became one of my favorite parts of this program.

Finally, I want to express my heartfelt appreciation to my friends and classmates at MIT. Thank you for your support, encouragement, and all the laughter along the way. You made this journey not only lighter but also unforgettable.

TABLE OF CONTENTS

1	Introduction.....	5
1.1	Motivation.....	5
1.2	Problem Statement	5
1.3	Project Goals and Expected Outcomes.....	6
2	State of the Practice	6
2.1	Vehicle Routing Problem (VRP).....	7
2.2	Inventory and Facility Rent Costs.....	7
3	Methodology	8
3.1	VRP Optimization Model.....	9
3.2	Inventory Replenishment Scenario Analysis along with Facility Rent Cost	12
3.2.1	Inventory Assumptions and Calculations	13
4	Results and Discussion	14
5	Conclusion	29

1 Introduction

1.1 Motivation

The middle mile in logistics is typically the part of a supply chain network where goods are moved from Distribution Centers (DCs) to Fulfillment Centers (FCs), without connecting directly to an end-customer location. For this project, the middle mile included deliveries from DCs to FCs and Stores, which are smaller in terms of volume processed but operate in a similar way as FCs for our sponsor company. In today's complex supply chain networks, balancing middle mile transportation costs and inventory costs is critical for operational efficiency and cost optimization. The challenge of finding the optimal transportation strategy and replenishment frequency between DCs and FCs or Stores is amplified by the inevitable tradeoff between transportation and inventory costs. Failing to make data-driven decisions can lead to increased costs, reduced service levels, and poor allocation of net working capital (NWC).

For our sponsor company, a world leader in electrical, communications, utility distribution, and supply chain services, maintaining such a balance across its operations is especially crucial. The company's EES (Electrical) and CSS (Datacom) business units utilize a Hub-and-Spoke distribution model, where DCs serve as hubs while FCs and Stores serve as spokes. Growth for our company has largely come from acquisitions, where newly acquired FCs are integrated into the existing delivery network. Their operational portfolio of products includes standard pick, pack, and ship processes, small sub-assembly operations at FCs, and functions performed by the Center of Excellence (COE). This project evaluated the effectiveness of the allocation model for pick-pack-ship products and identified potential areas for improvement. We optimized the replenishment frequency between hubs and spokes and lowered middle mile transportation costs.

Given the complexity of managing transportation and inventory costs across multiple regions, leveraging multi-stop milk runs becomes essential as higher replenishment frequency leads to more fragmentation. Our sponsor's current middle mile transportation strategy consists of a fleet mix of 55% parcel, 35% less-than-truckload (LTL), and 5% full-truckload (FTL). As part of this project, they expected us to optimize the network by utilizing FTL for middle mile transportation while developing replenishment policies that allow for a more cost-effective supply chain operation for the company. This is especially important as the company continues to acquire other businesses and needs an unbiased solution to help manage its complex inventory profile and SKU portfolio.

The sponsor company provided the assignment of each DC to its designated FCs and Stores, along with 12 months of data on their sales history for us to understand the demand to be fulfilled. To optimize transportation, inventory and facility-related costs, the company shared with us their transportation cost structure, inventory costs per unit and facility rent costs per square feet for DCs, FCs and Stores. By organizing and analyzing the data provided, with a focus on balancing transportation efficiency and inventory management, this project created a solution that reduced costs across the company's supply chain network.

1.2 Problem Statement

With rising transportation and inventory costs, the sponsor company's key objective is to develop an optimal middle mile transportation strategy and replenishment policy between their 7 hubs (DCs) and 176 spokes (FCs and Stores) covering a portfolio of 199 SKUs. For their pick-pack-ship products, each FC or Store can only receive inbound shipments from one designated DC. The current middle mile transportation model used by our sponsor company consists of 100% direct shipments, with each vehicle delivering to only one spoke (FC or Store) per trip. This leads to inefficiencies in balancing transportation costs with inventory levels, resulting in tied up NWC. The sponsor also

raised concerns that the current replenishment schedules are suboptimal, leading to excess inventory and high holding costs.

In this capstone we designed a methodology that defined the optimal replenishment frequency and transportation strategy, leveraging a mix of direct shipments and multi-stop milk runs. This methodology was tailored to regional demand patterns, with the goal of reducing NWC and lowering transportation and inventory costs. By implementing these optimized replenishment policies, the sponsor company expected to make more informed, data-driven decisions that enhance the efficiency and effectiveness of its supply chain operations.

In that context, the major questions answered in this capstone are:

1. What is the optimal replenishment frequency from each DC to its designated FCs and Stores to reduce the combined cost of transportation and inventory?
2. With the selected replenishment frequency, what is the optimal routing from each DC to its designated FCs and Stores to reduce the total cost of transportation?

1.3 Project Goals and Expected Outcomes

The primary objective of this project was to provide the sponsor company with a methodology to determine the optimal replenishment frequency while minimizing costs. The key deliverable to the company was a model that allows them to:

1. Find the optimal replenishment frequency and routing from each DC to its designated FCs and Stores
2. Compute the optimized middle mile transportation costs
3. Update the optimal solution based on latest demand.

Upon implementing this model, the sponsor company is expected to effectively integrate the scattered hubs and spokes of the various organizations acquired thus far, thereby optimizing the overall middle mile delivery network cost while maintaining its service level.

2 State of the Practice

The focus of our capstone is to provide our sponsor company with a way to find the optimal balance between transportation costs and inventory costs to minimize their overall supply chain costs. It is common for different functions within a company to desire opposite implementations on inventory management – one may ask for frequent replenishments and minimal inventory levels to reduce inventory holding costs, while another may aim to lower transportation costs by proposing to stock more materials and replenish less frequently. We approached the problem of minimizing the supply chain costs of our sponsor company by combining transportation, inventory and facility rent costs into a single objective.

To establish a robust foundation for our methodology, we conducted a comprehensive review of the literature on the vehicle routing problem (VRP), which provided insights into optimizing transportation logistics. Additionally, we examined regulatory and operational constraints relevant to the supply chain context. Our approach leverages mixed integer linear programming (MILP) techniques to formulate a mathematical model for cost optimization. Furthermore, we analyzed literature on inventory management and facility rent costs, which highlights the impact of replenishment frequency on cycle and safety stock levels. This theoretical framework informed the development of our model and ensured alignment with established supply chain optimization principles.

2.1 Vehicle Routing Problem (VRP)

The VRP is a cornerstone in logistics and supply chain management, addressing the optimal routing of vehicles to serve a set of customers while minimizing costs. Within VRP, the milk-run concept represents a specific variant where a single vehicle serves multiple suppliers or customers in a loop, optimizing goods transportation. To inform the methodology of this capstone project, we explored current practices in solving the VRP, with the goal of optimizing deliveries from DCs to FCs and Stores.

The most common VRP is classical VRP, and it focuses on determining the most efficient routes for vehicles delivering goods. Each vehicle is assigned a single route, possesses uniform characteristics, and operates from a central depot. The primary objective is to minimize the overall cost of transportation while ensuring that every customer (in our case FCs and Stores) is served exactly once by a vehicle, all routes begin and end at the depot, and vehicle capacity constraints are adhered to. Over time, the classical VRP has evolved to accommodate various practical considerations and complexities, giving rise to numerous variants that better reflect real-world scenarios.

VRP is so classic that there are over 1 million results related to it in Google Scholar. To understand VRP from a high-level perspective, we started by reviewing an overview of VRP and its variants provided by Braekers et al. (2015). Of the various scenarios and characteristics listed in Braekers et al. (2015), we determined that our capstone project is a VRP with deterministic demand, single period, no time window, a single depot (a spoke can receive goods from only one hub), and capacitated vehicles.

VRPs are formulated as MILPs and can be approached using exact and heuristic methods (Zhang et al., 2022). They often rely on heuristics methods because real-world applications of VRP are highly complex. For the sponsor's project, notable challenges include managing multiple FCs and Stores, handling split orders across these locations, and adhering to regulatory and operational constraints, all of which add layers of complexity to the optimization process. Nevertheless, with minor adjustments, we believed that classical VRP formulation would be sufficient to capture the problem setting of this capstone.

A significant challenge for the sponsor is adhering to regulatory frameworks such as the U.S. Federal Motor Carrier Safety Administration (FMCSA) regulations, which restrict drivers to 11 hours of driving within a 14-hour duty window. Factoring in time for loading and unloading at multiple stops, the effective operational window narrows to approximately 10 hours. Careful scheduling is required to ensure regulatory compliance while meeting delivery timelines. Additionally, trucks operate on a strict schedule, typically between 3 a.m. and 7 p.m., Monday through Friday.

To address these regulatory and operational challenges, as well as the broader complexities of real-world routing, MILP seemed the best approach for this capstone project to get started with the analysis. It is a mathematical optimization technique widely used for solving complex decision-making problems in various fields, including logistics, supply chain management, energy systems, and manufacturing. It extends linear programming (LP) by allowing some or all the decision variables to take integer values, enabling the modeling of discrete choices and logical constraints.

2.2 Inventory and Facility Rent Costs

Inventory carrying costs are a key part of supply chain management, covering expenses like capital tied up in cycle stock inventory, safety stock inventory and facility rent costs. As replenishment frequency decreases from daily to weekly, bi-weekly, or monthly, cycle stock and safety stock levels at FCs and Stores increase.

Facility rent costs are fixed monthly expenses for DCs, FCs, and Stores. When inventory levels rise due to lower replenishment frequencies, it leads to higher holding costs and creates a need for more facility space, thereby increasing rent expenses. The combination of inventory carrying costs and facility rent costs plays a major role in the overall efficiency and cost of the supply chain.

Current practices in supply chain management emphasize the need for integrated approaches to balance these cost components. Chopra and Meindl (2016) highlight the importance of aligning inventory and transportation decisions to optimize replenishment policies, noting that facility capacity constraints must be considered to minimize total costs. Their work discusses the trade-offs between holding costs and transportation expenses, a key consideration in our analysis of the sponsor's middle mile network. Similarly, Rushton et al. (2017) advocate for modeling of facility rent costs, arguing that these costs, often treated as fixed, vary with storage requirements driven by inventory levels. This perspective is particularly relevant for hub-and-spoke networks, where inventory policies directly impact facility utilization and costs.

Additionally, Zipkin (2000) provides a comprehensive framework for understanding inventory dynamics, detailing the behavior of cycle stock and safety stock in single-stage networks with demand points like FCs and Stores. Zipkin's (2000) insights into how replenishment frequency affects inventory costs and service levels inform contemporary approaches to designing cost-effective supply chain strategies. These sources collectively emphasize the state of the practice in jointly evaluating transportation, inventory, and facility rent costs to achieve financial optimization in supply chain networks.

Bearing in mind the trade-offs between inventory costs and transportation costs highlighted in the literature, our capstone project explored different replenishment frequency scenarios to compare the total costs for each DC network, ultimately identifying the optimal replenishment frequency that minimizes the sponsor company's overall cost and net working capital.

3 Methodology

To achieve the goals of this project, we developed mathematical functions to find optimal routes for each DC to its designated FCs and Stores, implemented the mathematical functions in a high-performance VRP solver package, and conducted an inventory replenishment scenario analysis.

Based on the sponsor company's objective and the literature we reviewed, we determined the best methodology to find the optimal solution for inventory replenishment policy and middle mile transportation for each DC.

We considered both direct shipments and milk-run deliveries from each DC to FCs and Stores (Figure 1). To reflect actual routes and traffic patterns for trucks, we imported Google Maps' Distance Matrix API to our notebook in Google Colab to compute the distance and average travel time between nodes in the network. For inventory replenishment, we ran multiple scenarios to determine the optimal replenishment frequency decision for each DC to its designated FCs and Stores. Bakir et al. (2021) applied MILP to find the optimal solution for flow planning consolidating trucking networks. Likewise, we utilized MILP to optimize the middle mile transportation network and extended the model to include inventory and facility rent considerations, enabling a comprehensive evaluation of total supply chain costs.

Figure 1. Middle Mile Network for Each DC of Our Sponsor Company



3.1 VRP Optimization Model

Starting with a relatively simple model, we decided to focus on constructing the middle mile network optimization model first. Later, after we confirmed the inventory cost calculations, including service level, ordering costs, holding costs, etc., with the sponsor company, we ran multiple scenarios on different replenishment frequencies. The basic structure of our VRP model, consisting of sets and parameters, decision variables, an objective function, and constraints of our MILP in the early stage without considering inventory levels, is as follows:

Sets and Parameters:

- N : Set of nodes (DC 0, FCs and Stores 1, 2, ..., n)
- A : Set of arcs between nodes (DC-FC, DC-Store, FC-Store, FC-FC, Store-Store connections)
- d_i : Demand of FC/ Store i (in pounds), $i \in N \setminus \{0\}$
- Q : Truck capacity (38,000 pounds)
- T : Total number of trucks available
- h_{ij} : Travel time (hours) between nodes i and j , $(i, j) \in A$
- c_{ij} : Transportation cost for traveling between nodes i and j , $(i, j) \in A$
- H : Maximum operational hours per day, including loading and unloading (10 hours)

Decision Variables:

- x_{ij}^k : Binary variable. Equals 1 if truck k travels from node i to j , and 0 otherwise, for $(i, j) \in A$, $k = 1, 2, \dots, T$

- q_{ij}^k : Continuous variable. Represents the weight (in pounds) delivered by truck k on arc (i, j) , for $(i, j) \in A, k = 1, 2, \dots, T$

Objective Function (minimize the total transportation cost):

$$\text{Minimize } \sum_{k=1}^T \sum_{(i,j) \in A} c_{ij} x_{ij}^k$$

Constraints requested by our sponsor company:

1. Demand Fulfillment

The total weight delivered to each customer must meet their demand (split deliveries allowed due to demand per destination node could exceed truck capacity):

$$\sum_{k=1}^T \sum_{j \in N: (i,j) \in A} q_{ij}^k = d_i, \forall i \in N \setminus \{0\}$$

2. Truck Capacity

The total weight carried by a truck on its route cannot exceed its capacity:

$$\sum_{(i,j) \in A} q_{ij}^k \leq Q, \forall k = 1, 2, \dots, T$$

3. Time Constraint

The total travel time for each truck must not exceed the daily operational limit:

$$\sum_{(i,j) \in A} h_{ij} x_{ij}^k \leq H, \forall k = 1, 2, \dots, T$$

4. Arc Consistency

Goods can be delivered only if a truck travels along the corresponding arc:

$$q_{ij}^k \leq Q * x_{ij}^k, \forall (i, j) \in A, k = 1, 2, \dots, T$$

5. Round Trip Constraint for Truck Flow

Each truck must start and end its route at the DC:

$$\sum_{j \in N: (0,j) \in A} x_{0j}^k = \sum_{j \in N: (j,0) \in A} x_{j0}^k, \forall k = 1, 2, \dots, T$$

6. Subtour Elimination Constraint

Use the Miller-Tucker-Zemlin (MTZ) constraints to eliminate subtours:

$$u_i^k - u_j^k + n * x_{ij}^k \leq n - 1, \forall i, j \in N \setminus \{0\}, k = 1, 2, \dots, T, i \neq j$$

u_i^k : auxiliary variable representing the position of customer i in truck k 's route

u_j^k : auxiliary variable representing the position of customer j in truck k 's route

The MTZ constraint is a mathematical formulation used to eliminate subtours in routing problems such as the VRP and the Traveling Salesman. Subtours are routes that only include nodes other than the depot, which is the DC in our case. Without this constraint, the model might generate solutions that violate our rule of starting and ending each route at the DC. It is a concept that has been widely adapted in routing problems. As early as 1991, Desrochers and Laporte published an article with a focus on the improvements and extensions of the MTZ constraint, which has been cited over 700 times.

7. Truck Usage Limit

The number of trucks used cannot exceed the total number of trucks available:

$$\sum_{k=1}^T \sum_{j \in N: (0,j) \in A} x_{0j}^k \leq T$$

As we implemented the above methodology in Google Colab, we encountered some obstacles. First, before we added the time constraint to our optimization model, we calculated the distance and duration between nodes using Google Maps Distance Matrix API. We discovered that half of the spokes (FCs and Stores) require over 5.5 hours for any truck to deliver from the designated DC and thus over 11 hours for a round trip without multiple stops. In other words, if we added the time constraint on top of the round-trip constraint, milk runs would likely not occur for at

least half of the spokes. Therefore, after discussions with the sponsor company confirming that their drivers can deliver to spokes and then return to the depot the next day, we removed the time constraint to allow for possible milk runs, which can be performed in multiple days.

Second, we attempted to solve the optimization for each DC by using Gurobi Optimizer in Google Colab, but it took over 2 hours for the model to run for the DC with the least spokes due to the excessive number of variables, nodes, and constraints. As for the DCs serving over 15 spokes, the models would simply crash. With that in mind, we realized an alternative solution had to be sought.

As mentioned in 2.1, VRP often relies on heuristic solutions, so we explored the concept of hybrid genetic search for the capacitated vehicle routing problem (Vidal, 2022), the latest state of the art, and PyVRP, the latest high-performance VRP solver package (Wouda et al., 2024). PyVRP is a heuristic model that can produce extremely close results to those of a Gurobi model within a short period of time. In our case, PyVRP reduced the model solving time to less than two minutes, even for the DC serving the largest number of FCs and Stores. Thus, we decided to implement the hybrid genetic search for the capacitated VRP by using PyVRP instead of Gurobi to solve our problem.

We first calculated the number of FTLs required to serve each spoke by dividing the demand of each spoke by the truck capacity (38,000 pounds). Then the remaining demand of each spoke, which was less than a full truckload, was fed into PyVRP to generate routes that allow for milk runs.

3.2 Inventory Replenishment Scenario Analysis along with Facility Rent Cost

Adding inventory levels and facility rent costs into consideration, we conducted multiple replenishment scenarios to evaluate their performance by seeing which one results in the lowest costs per unit of time. For instance, we started by analyzing what happens if we keep the replenishment frequency at once a week, assessing the resulting inventory levels, facility space utilization, and overall costs. Similarly, we evaluated the impact of increasing the frequency to twice a week, comparing the shift in inventory levels, facility rent and transportation costs. By simulating these scenarios, we aimed to identify the optimal replenishment frequency from each DC to its designated FCs and Stores, achieving the lowest overall costs per unit of time while maintaining desired service levels and operational efficiency.

To run multiple scenarios for each DC, we used the 12-month historical sales data provided by the sponsor company, along with additional information on inventory carrying costs and facility rent costs. For this project, seasonality and SKU-level demand variability were not considered. Instead, all SKUs were treated uniformly based on average

demand, building a model that the sponsor company could later adapt by plugging in real-time customer order data or demand forecasts.

3.2.1 Inventory Assumptions and Calculations

To determine inventory levels, we considered the following assumptions:

- Monthly Demand Variability = 20%
- Holding Cost = 12% of the product value
- Ordering Cost = \$15 per order line
- Service Level = Targeted at 95%, corresponding to a Z-score of 1.64.

Using this data, inventory levels were calculated by first determining Cycle Stock and Safety Stock for a particular replenishment frequency.

$$\text{Cycle Stock} = \text{Demand}/2 ,$$

$$\text{Safety Stock} = Z \times \sigma_d \times L$$

where:

- $Z = 1.64$ (for a 95% service level)
- σ_d = Standard deviation of demand (calculated as Demand Variability $\times$ Average Demand)
- L = Lead time (assumed as 1 day for daily replenishment, 7 days for weekly replenishment, 14 days for bi-weekly and 30 days for monthly)

By summing Cycle Stock and Safety Stock, we derived the Total Inventory Levels for each replenishment scenario. This framework then allowed us to compare various replenishment strategies, ensuring an optimal balance between transportation and inventory costs while maintaining service level targets.

Other than a recommendation on the routing and replenishment policy for each DC to its designated FCs and Stores based on the historic data they provided, the sponsor company asked for a model that they could utilize in the future as demand fluctuates. Therefore, the way we constructed our code allows the routing, number of fleets required, and transportation costs to update upon the latest data loaded to the model.

After calculating the total inventory cost for each replenishment scenario, it was crucial to assess whether the demand points can accommodate the recommended inventory volumes. For instance, FCs or stores need to have sufficient space to store the suggested cycle stock and safety stock. To account for space constraints in the model, a

"\$ per square foot" cost was added to the total cost. This cost reflected the additional space required to manage increased safety and cycle stock levels and was included as part of the total inventory cost.

4 Results and Discussion

This capstone project delivers the lowest middle mile logistics costs, consisting of transportation and inventory costs, for our sponsor company. To evaluate which replenishment frequency delivers the best solution, we generated different scenarios to make comparisons: Daily Replenishment Scenario, Weekly Replenishment Scenario, Bi-Weekly Replenishment Scenario, and Monthly Replenishment Scenario.

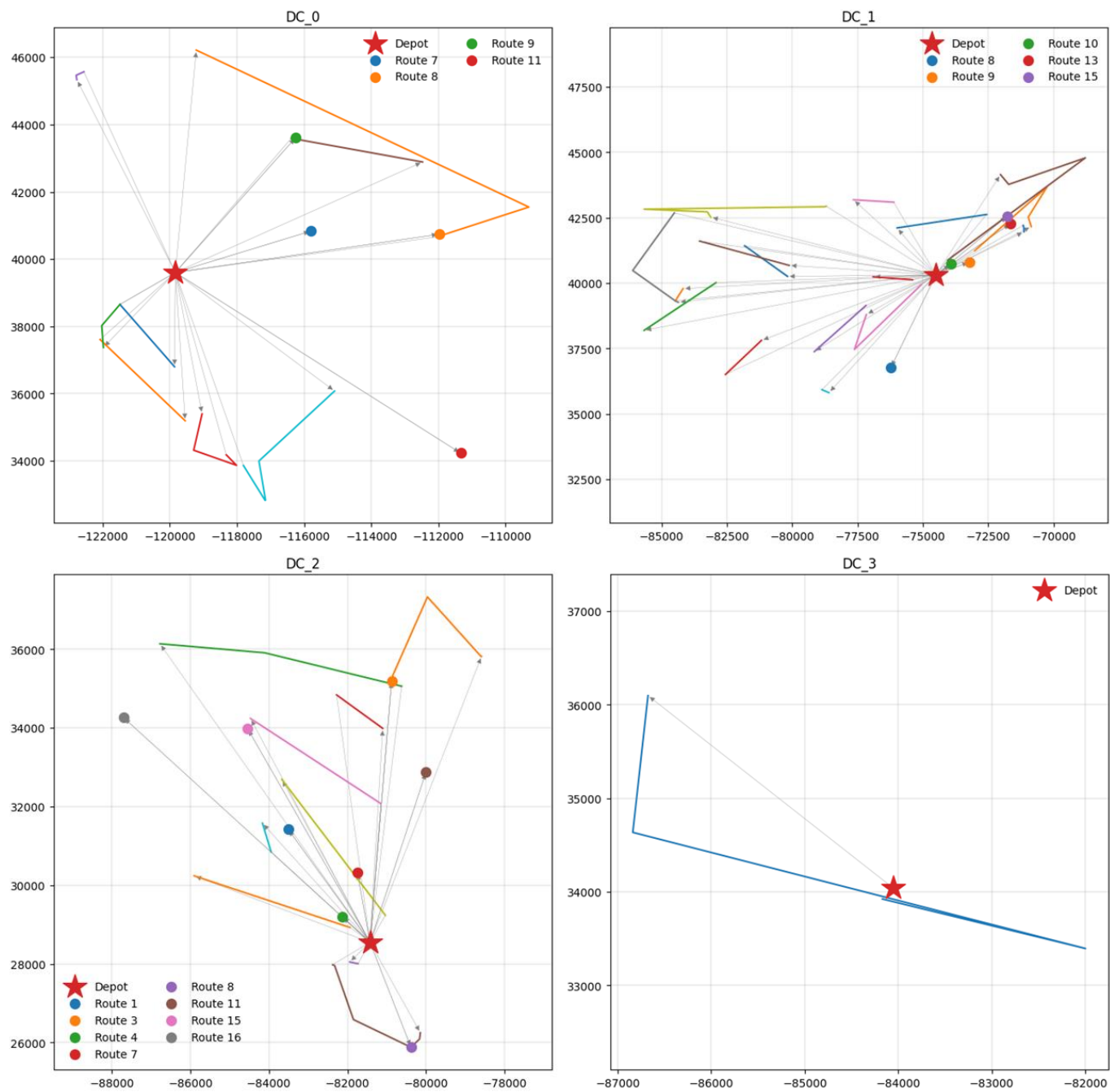
Scenario 1: Daily Replenishment Scenario: Compared to 100% direct shipments, the total distance traveled reduced by 14.05% and the number of trucks (NWC) decreased by 98 each day as milk runs were incorporated in the routes. Detailed changes after incorporating milk runs for each DC are listed in Table 1. Visualization of the routes incorporating milk runs is displayed in Figure 2.

Table 1. Transportation for Each DC Set – Daily Replenishment

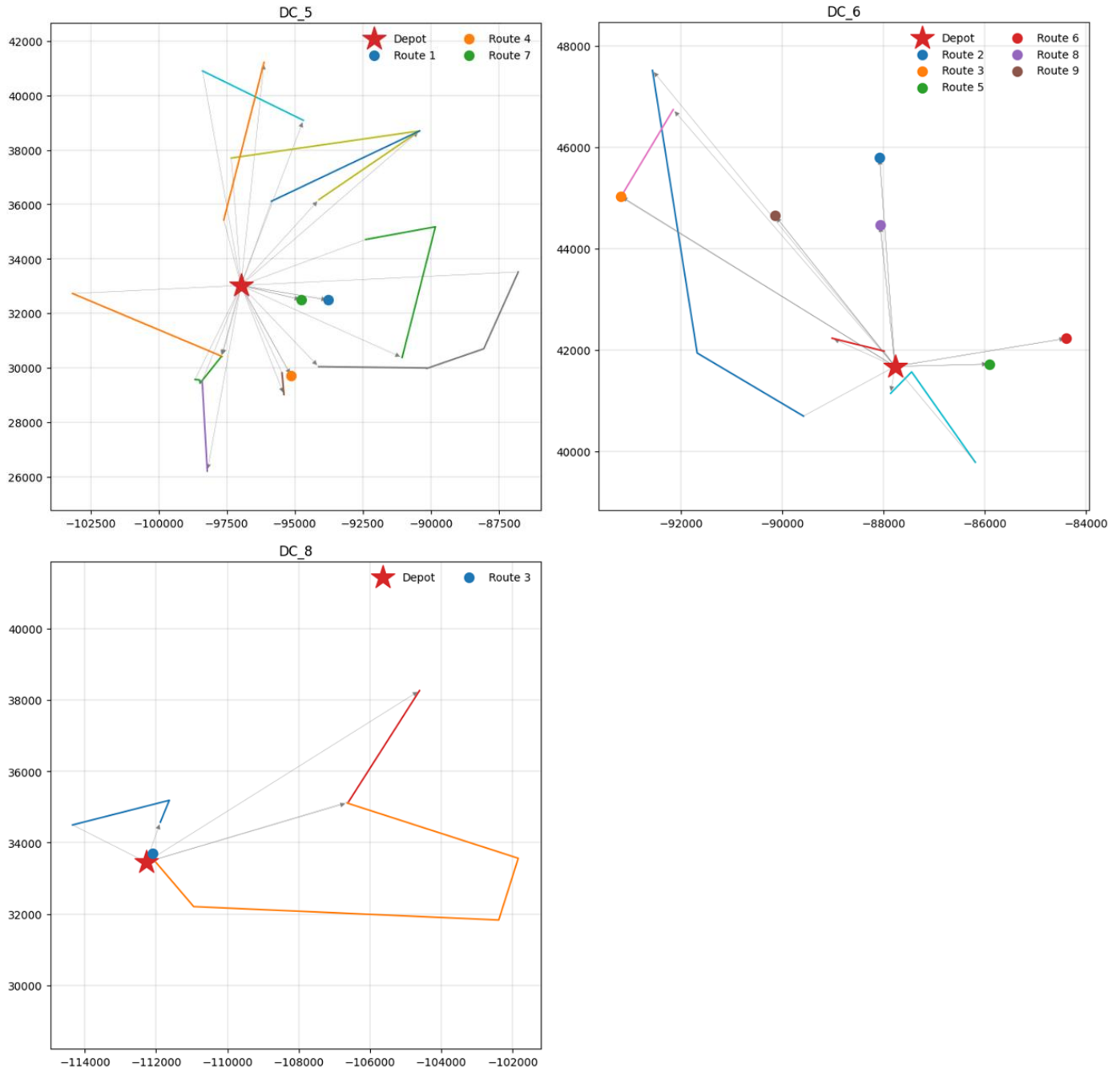
DC	DC_0	DC_1	DC_2	DC_3	DC_5	DC_6	DC_8
Change in Distance Traveled	-17.51%	-21.44%	-6.24%	-24.78%	-18.78%	-6.69%	-20.28%
Change in Number of Trucks	-19	-27	-17	-3	-19	-6	-7
Scaled Demand in Pounds	35	41	100	2	29	23	11

The percentage decrease of distance traveled for each DC set was deeply influenced by the demand the DC was fulfilling and the decrease in number of routes, equivalent to the decrease in number of trucks, after incorporating milk runs. Although the number of routes required was only reduced by 3 for DC_3, its percentage decrease of distance traveled turned out to be the highest among all DCs in the daily replenishment scenario because the demand it served was very low compared to other DCs. In contrast, DC_2 served the most demand, while the number of routes required only decreased by 17, making its percentage decrease of distance traveled the lowest of all.

Figure 2. Routes of Each DC Set – Daily Replenishment¹



¹ Latitude and longitude values are shown in thousandths of a degree (e.g., 45000 = 45.000°).

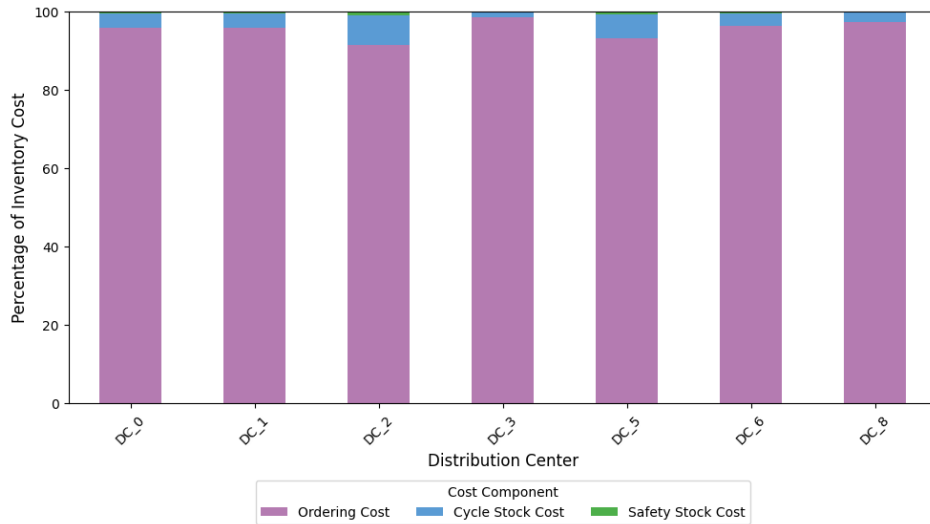


Impact of Daily Replenishment (Scenario 1) on Inventory Levels:

In Scenario 1, the analysis reveals that ordering cost is a significant contributor to overall daily inventory costs. Daily replenishment helps reduce cycle stock and safety stock, which lowers holding costs. However, because orders are placed more frequently, the cost of ordering goes up. Ordering costs are mainly tied to the workforce needed to create and manage purchase orders (POs). Since daily replenishment means generating many POs every day, it increases labor costs. This shows that while daily replenishment saves on holding costs, it adds to ordering costs,

creating a trade-off that needs careful management. The breakdown for all components for inventory costs is depicted in Figure 3.

Figure 3. Breakdown of Inventory Costs for Daily Replenishment



Scenario 2: Weekly Replenishment Scenario: Compared to 100% direct shipments, the total distance traveled reduced by 2.66% and the number of trucks (NWC) decreased by 82 each week as milk runs were incorporated in the routes. Detailed changes after incorporating milk runs for each DC are listed in Table 2. Visualization of the routes incorporating milk runs is displayed in Figure 4.

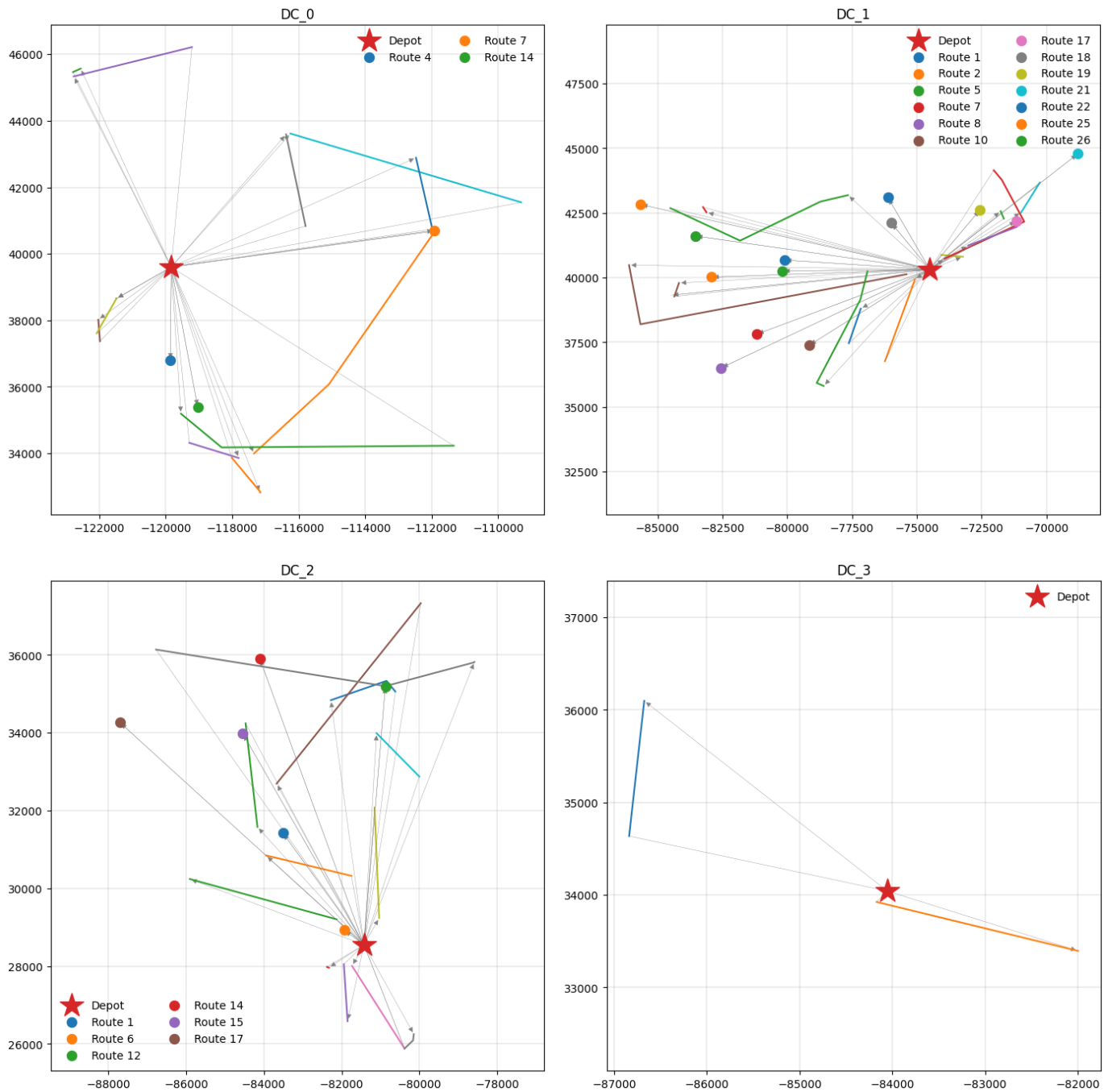
Table 2. Transportation for Each DC Set – Weekly Replenishment

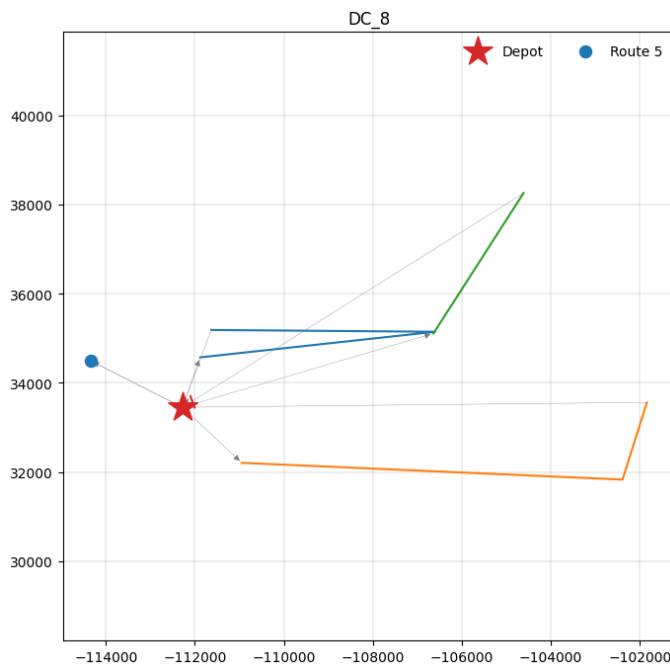
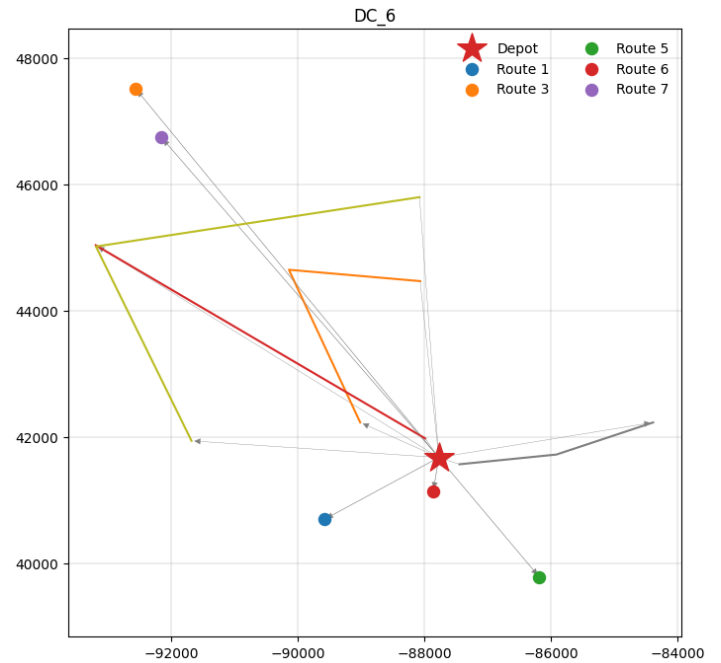
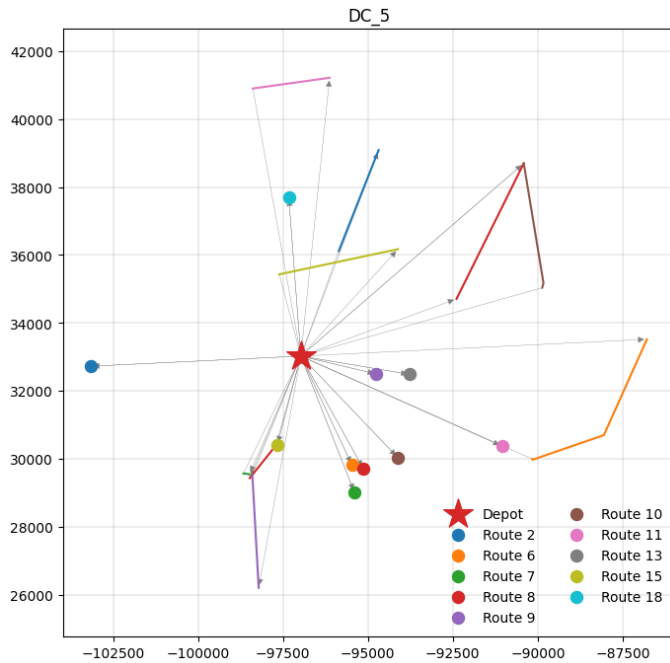
DC	DC_0	DC_1	DC_2	DC_3	DC_5	DC_6	DC_8
Change in Distance Traveled	-3.59%	-3.97%	-1.13%	-9.05%	-3.72%	-1.80%	-4.27%
Change in Number of Trucks	-16	-22	-16	-2	-13	-7	-6
Scaled Demand in Pounds	175	205	500	10	145	115	55

Similar to the daily replenishment scenario, DC_3 once again achieved the largest percentage decrease of distance traveled among all the DCs due to its low demand, and therefore low baseline distance traveled. Surprisingly, DC_6, instead of DC_2 resulted in the lowest percentage decrease after incorporating milk runs in the weekly replenishment scenario. Exploring in depth the distribution of the remaining demand that could not fill an FTL of the FCs and Stores served by DC_6 helped us realize the reason for this result. When a truck fulfills the demand of multiple FCs and Stores in one route, the total weight of these demand points must not exceed the truck capacity. Therefore, the routes of DC_6 in Figure 4 explained why the trucks could not simply visit FCs and Stores close in proximity when the total remaining demand of these locations exceeded the capacity. Considering the capacity

constraint, the milk runs in DC_6 was not the most efficient in minimizing distance traveled, and thus the relatively low percentage of decrease.

Figure 4. Routes of Each DC Set – Weekly Replenishment

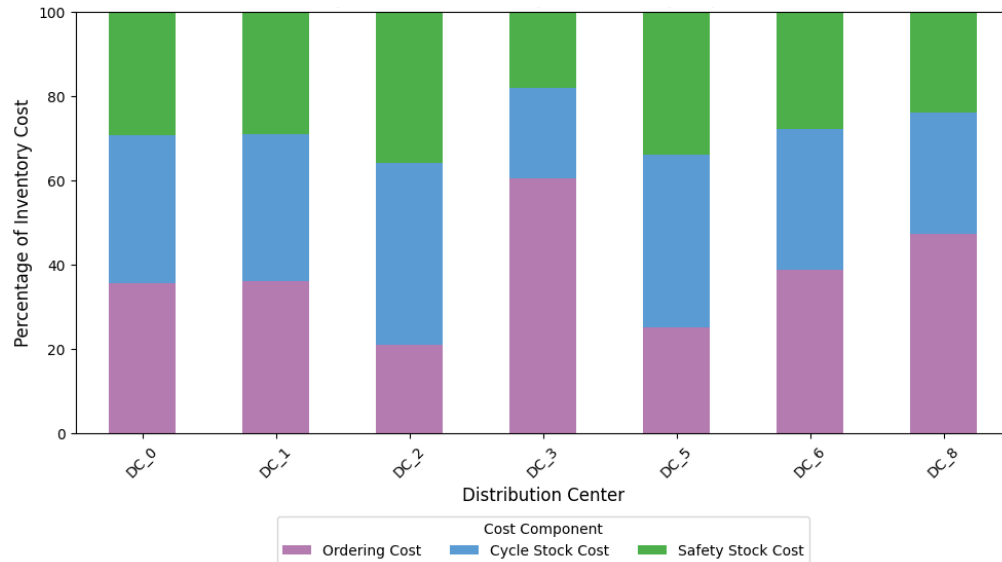




Impact of Weekly Replenishment (Scenario 2) on Inventory Levels:

In Scenario 2, ordering costs decrease to 54% with less frequent ordering, but cycle stock (24%) and safety stock (21%) costs increase. This trade-off between ordering and holding costs is depicted in Figure 5, highlighting the impact of order frequency on inventory cost distribution.

Figure 5. Breakdown of Inventory Costs for Weekly Replenishment



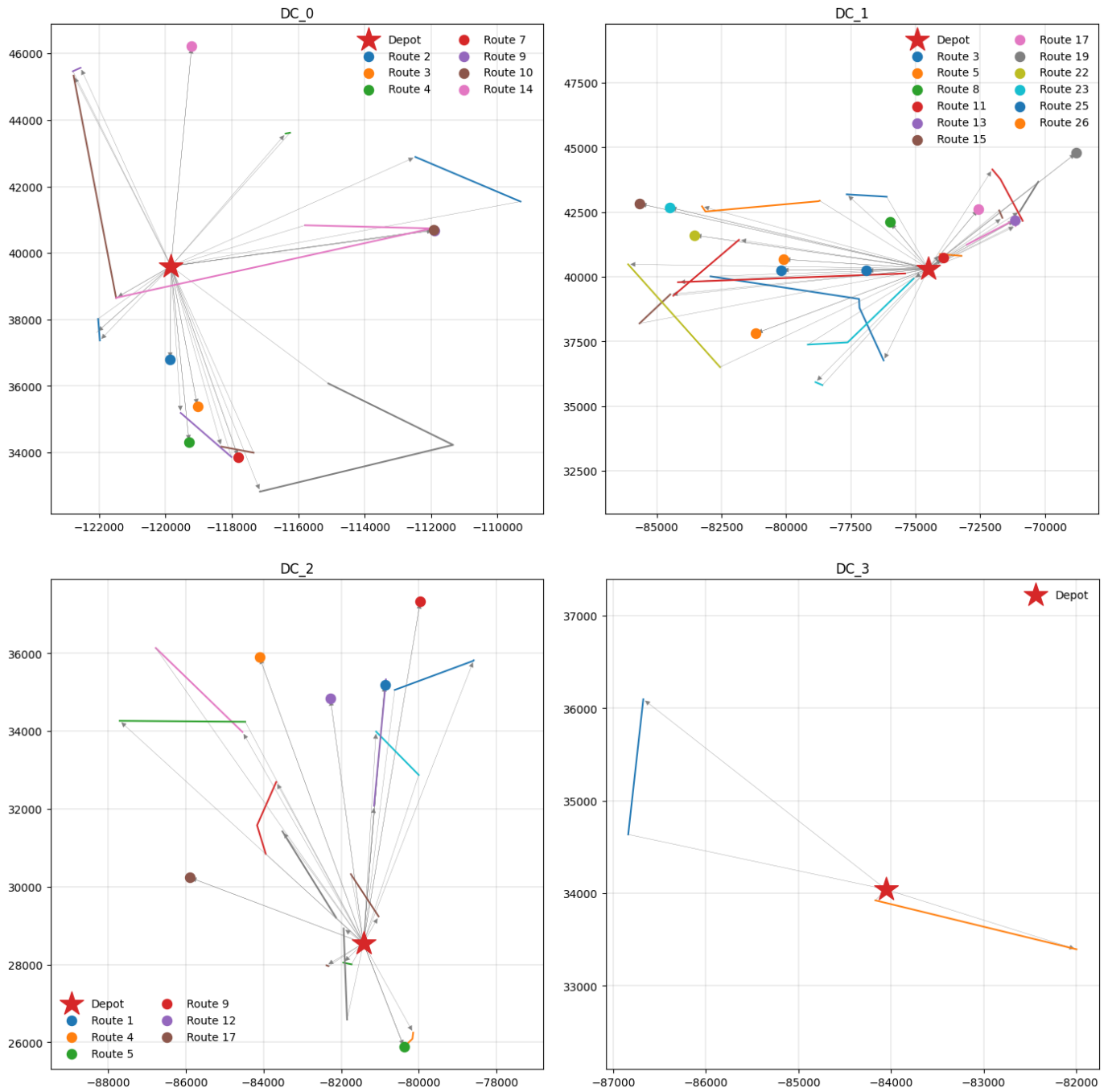
Scenario 3: Bi-Weekly Replenishment Scenario: Compared to 100% direct shipments, the total distance traveled reduced by 1.29% and the number of trucks (NWC) decreased by 81 every two weeks as milk runs were incorporated in the routes. Detailed changes after incorporating milk runs for each DC are listed in Table 3. Visualization of the routes incorporating milk runs is displayed in Figure 6.

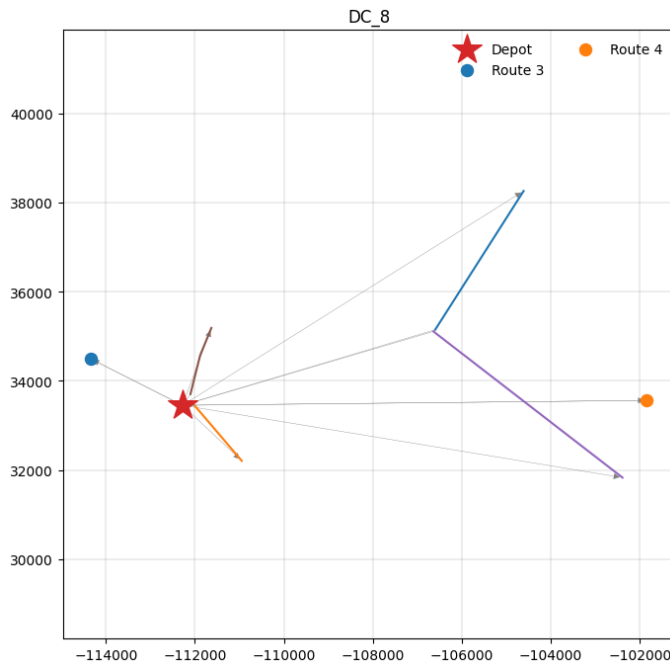
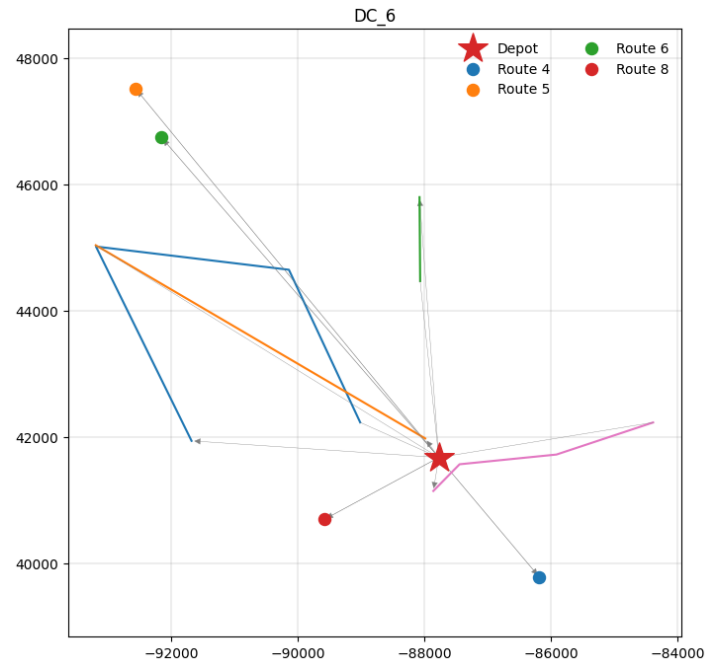
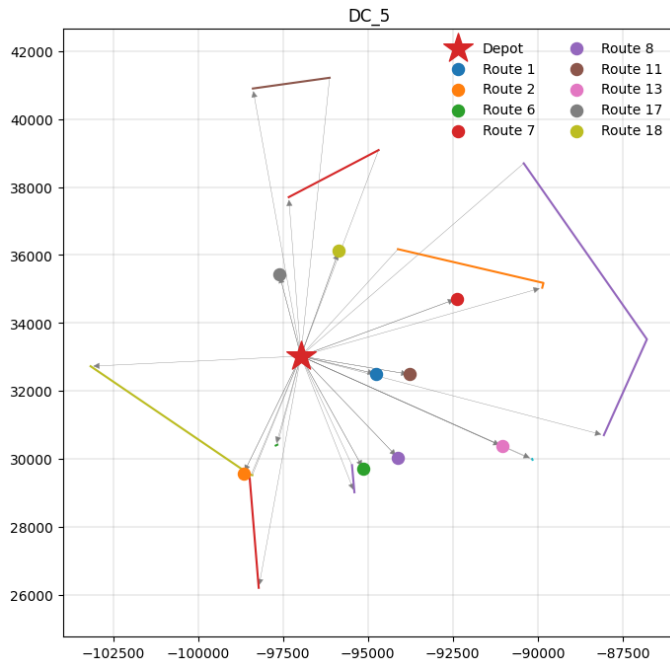
Table 3. Transportation for Each DC Set – Bi-Weekly Replenishment

DC	DC_0	DC_1	DC_2	DC_3	DC_5	DC_6	DC_8
Change in Distance Traveled	-1.56%	-2.00%	-0.59%	-4.98%	-2.06%	-1.00%	-1.32%
Change in Number of Trucks	-14	-22	-16	-2	-14	-8	-5
Scaled Demand in Pounds	350	410	1000	20	290	230	110

DC_3 yielded the largest percentage decrease of distance traveled as it did in daily and weekly replenishment scenarios, whereas DC_2 and DC_6 yielded the lowest. As explained in the previous scenarios, these results could be interpreted by the change in number of routes, baseline distance traveled based on demand, and distribution of the FCs and Stores served in one milk run considering truck capacity.

Figure 6. Routes of Each DC Set – Bi-Weekly Replenishment

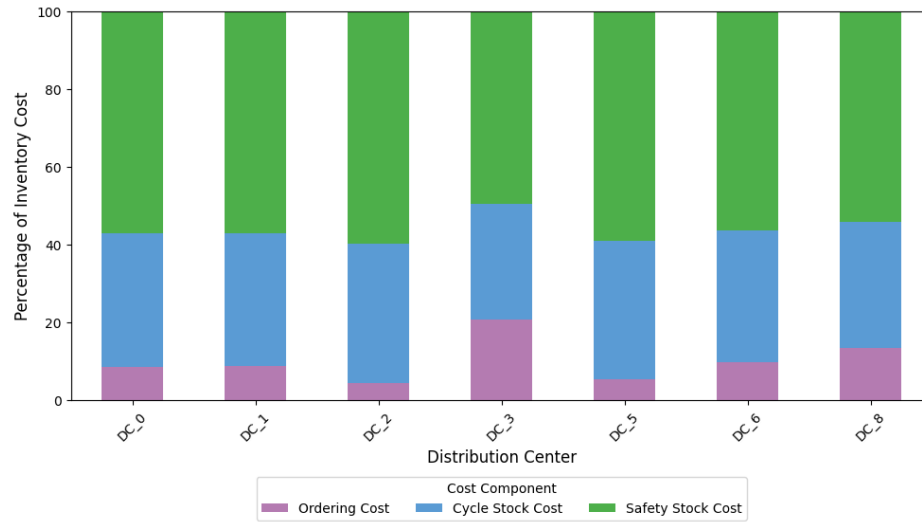




Impact of Bi-Weekly Replenishment (Scenario 3) on Inventory Levels:

With ordering frequency reduced to once every 3 weeks, ordering costs decreased. However, this frequency requires maintaining higher inventory levels, leading to an increase in safety stock and cycle stock. As a result, their contribution to total inventory costs rises, as illustrated in Figure 7.

Figure 7. Breakdown of Inventory Costs for Bi-Weekly Replenishment



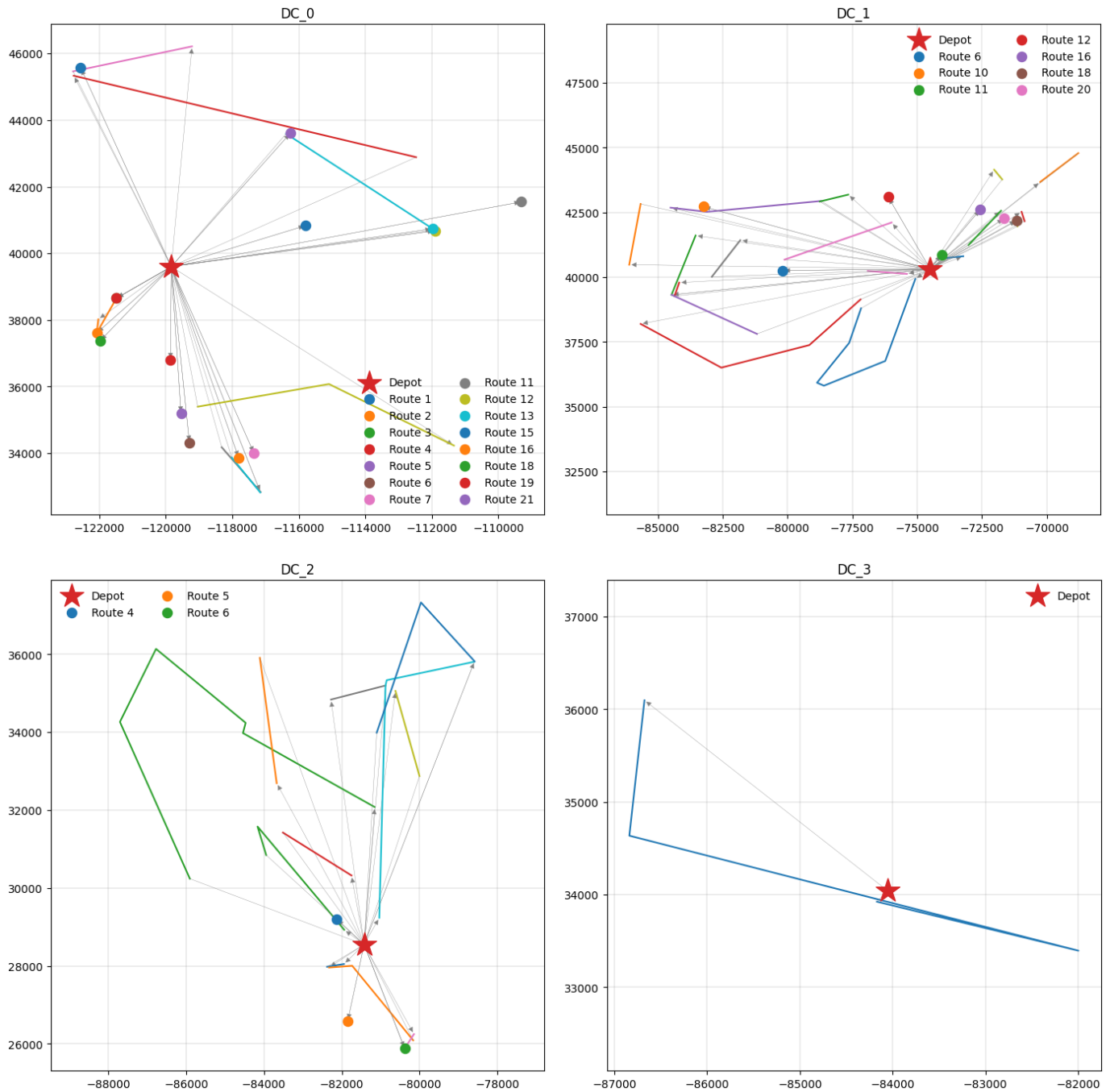
Scenario 4: Monthly Replenishment Scenario: Compared to 100% direct shipments, the total distance traveled reduced by 0.61% and the number of trucks (NWC) decreased by 80 every month as milk runs were incorporated in the routes. Detailed changes after incorporating milk runs for each DC are listed in Table 4. Visualization of the routes incorporating milk runs is displayed in Figure 8.

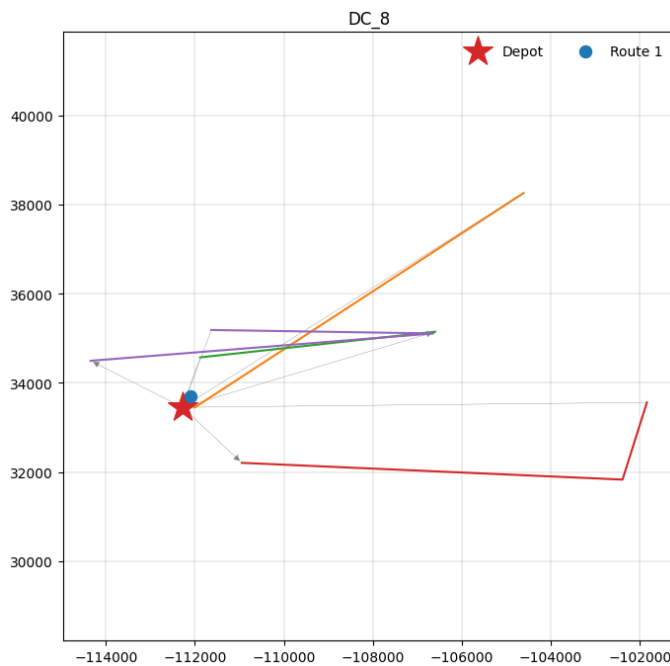
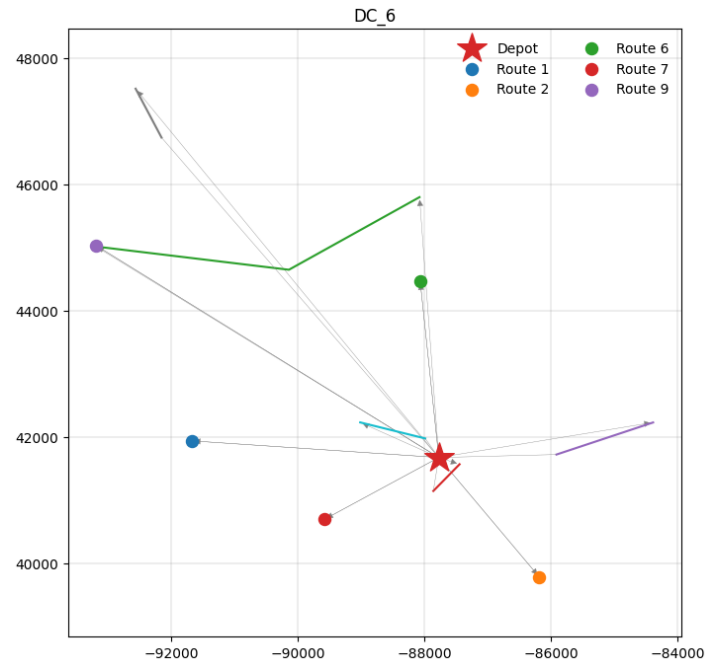
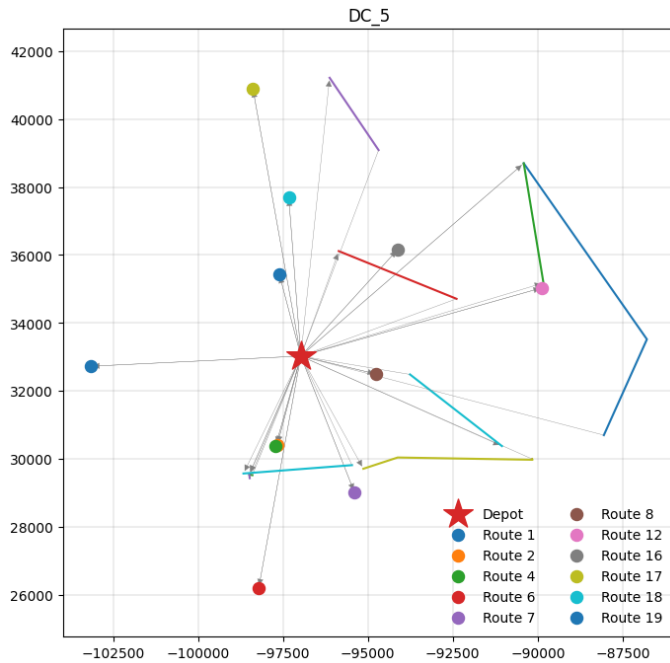
Table 4. Transportation for Each DC Set – Monthly Replenishment

DC	DC_0	DC_1	DC_2	DC_3	DC_5	DC_6	DC_8
Change in Distance Traveled	-0.50%	-1.10%	-0.37%	-2.62%	-0.75%	-0.58%	-0.75%
Change in Number of Trucks	-9	-24	-20	-3	-12	-6	-6
Scaled Demand in Pounds	758	888	2167	43	628	498	238

As we analyzed the cost saving from daily, weekly, bi-weekly, and monthly replenishment scenarios, we found that the percentage drop in overall distance traveled decreased as the replenishment frequency decreased. This could mainly be explained by the baseline distance traveled of the scenarios – the higher the replenishment frequency, the lower the total demand, thus the baseline distance traveled, the larger the percentage decrease of distance traveled could be achieved for each DC. In this scenario the results once again showed that DC_3, with the least demand, yielded largest percentage decrease of distance traveled, while DC_2 remained the least.

Figure 8. Routes of Each DC Set – Monthly Replenishment

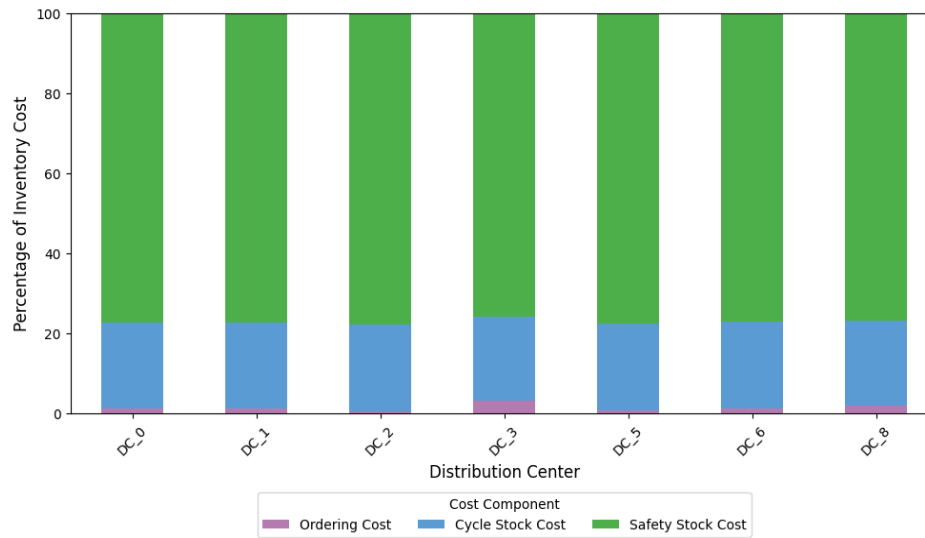




Impact of Monthly Replenishment (Scenario 4) on Inventory Levels:

Scenario 4 represents the lowest ordering frequency, with replenishment occurring only once a month. This frequency minimizes ordering costs but requires higher safety stock and cycle stock to maintain the desired 95% service level. As shown in Figure 9, ordering costs become negligible in the overall inventory cost structure, while the costs associated with safety stock become significant.

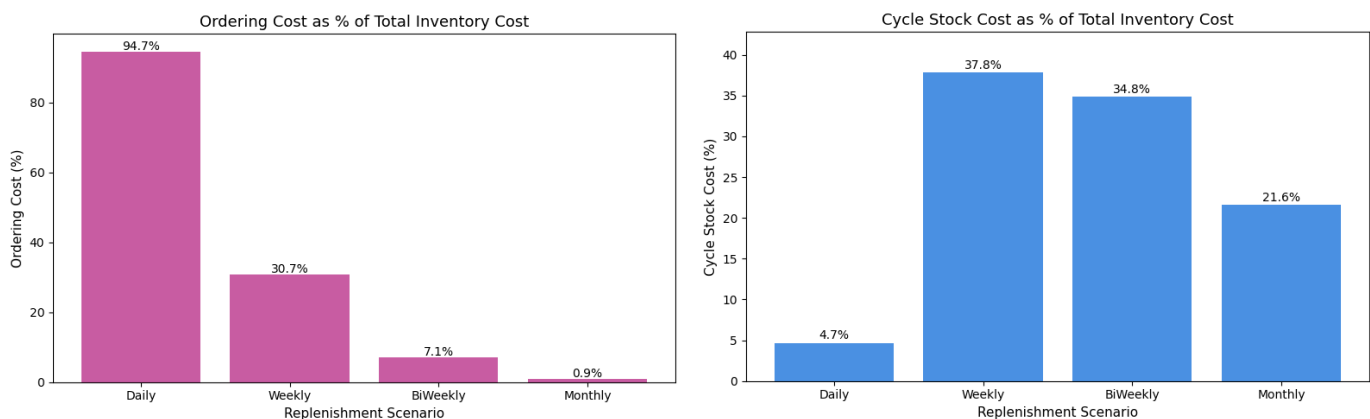
Figure 9. Breakdown of Inventory Costs for Monthly Replenishment

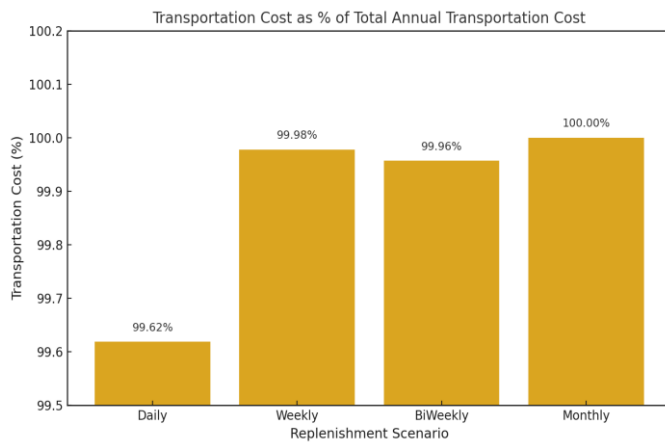
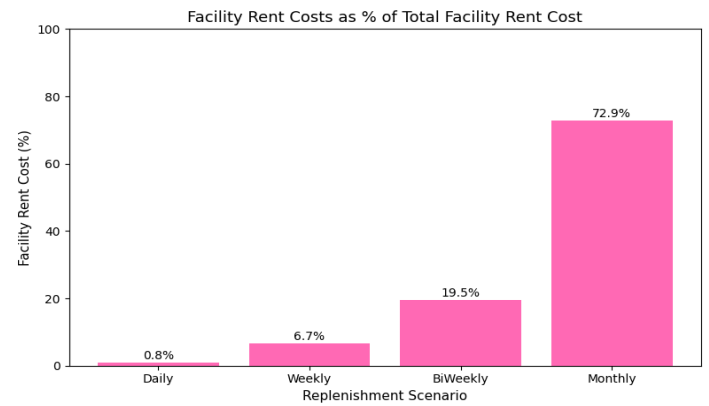
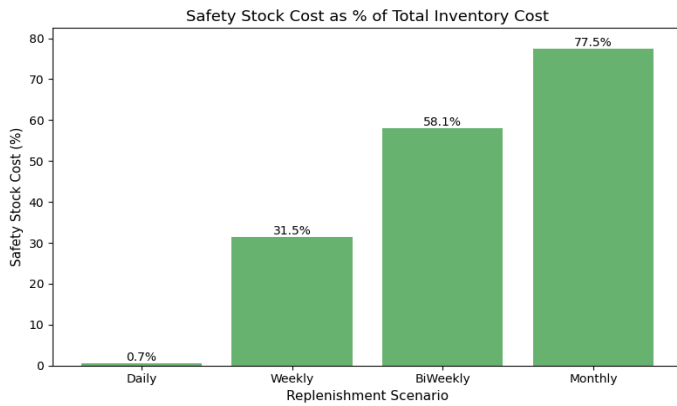


Summary of Inventory Cost, Facility Rent Cost and Transportation Cost across Various Replenishment Frequencies

The inventory costs (ordering cost, cycle stock cost, and safety stock cost), facility rent costs and transportation costs were analyzed across various replenishment models. Figure 10 illustrates the contribution of these costs to the total cost for daily, weekly, bi-weekly, and monthly replenishment scenarios. As replenishment frequency decreased from daily to monthly, ordering costs declined, while safety stock costs increased. Although cycle stock costs increased in absolute terms, its contribution to total inventory costs remained lower compared to safety stock costs. Thus, with increased replenishment frequency, safety stock cost emerges as the primary driver of total inventory cost.

Figure 10. Impact of Replenishment Frequencies on Ordering Cost, Safety Stock Cost, Cycle Stock Cost, Facility Rent Cost, and Transportation Costs





When the combined transportation costs, inventory costs, and facility rent costs for all DCs were evaluated across the four replenishment scenarios (see Table 5), the analysis reveals that in most cases total costs are minimized under a weekly replenishment schedule (except for DC_2). Daily, bi-weekly, and monthly replenishment resulted in costs 6% - 68% higher than those of the weekly option, suggesting that weekly replenishment offers the best balance between transportation and inventory costs (Figure 11).

Figure 11. Impact of Replenishment Frequencies on Total Costs

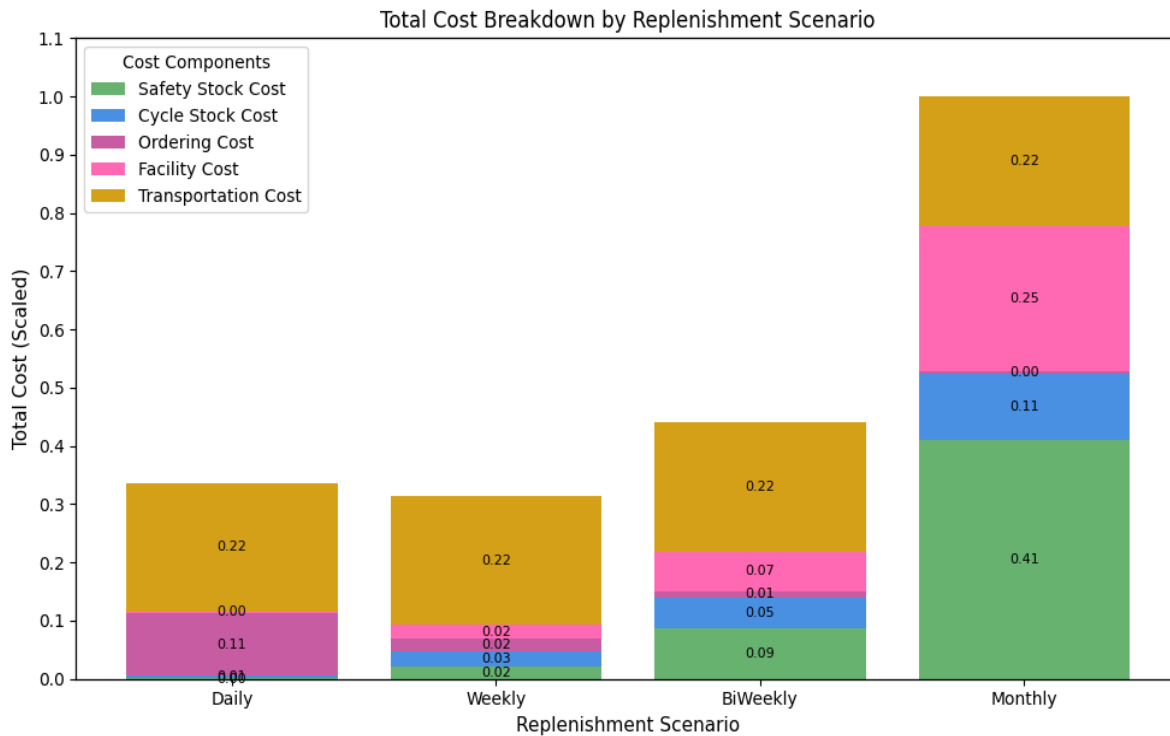


Table 5. Total Transportation Cost, Inventory Cost, and Facility Rent Cost Comparison Across all Scenarios

DC	DC_0	DC_1	DC_2	DC_3	DC_5	DC_6	DC_8	Total
Daily	37.20%	33.87%	35.30%	62.72%	22.91%	40.36%	47.79%	33.70%
Weekly	33.01%	29.34%	36.37%	41.68%	22.59%	34.62%	37.81%	31.46%
Bi-weekly	45.09%	42.07%	48.27%	50.67%	36.90%	46.27%	48.48%	43.99%
Monthly	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%

It should be noted that these inventory figures may change if any assumptions or calculation parameters are modified. A sensitivity analysis was performed by varying demand variability to 10% and 30% ($\pm 10\%$ from the baseline of 20%). At the baseline of 20% demand variability, the results confirmed that weekly replenishment remained the most cost-effective option in most cases. However, for DC_2, daily replenishment proved more cost-effective.

These findings inform to optimize replenishment across the network by implementing weekly replenishment for DC_0, DC_1, DC_3, DC_5, DC_6, and DC_8, where total costs are minimized. For DC_2, daily replenishment has the lowest total cost and thus it is advisable to implement daily replenishment for DC_2. Through implementing the

optimal replenishment frequency and routing strategy for each DC set, the sponsor company can achieve total cost reduction and free up net working capital for the middle mile network.

5 Conclusion

This project successfully optimized the middle mile transportation and inventory replenishment strategy for the sponsor company, balancing transportation costs, inventory holding costs, and facility rent costs. By modeling multiple replenishment scenarios and integrating vehicle routing solutions through PyVRP for each scenario, we calculated the total cost for each option. The analysis showed that weekly replenishment offered the most cost-effective balance, minimizing middle mile logistics costs in most cases (except DC_2). While higher replenishment frequencies reduced holding costs, they led to higher ordering costs, whereas less frequent replenishments increased inventory and facility rent costs. Our recommendations provide a flexible model that the sponsor can update based on future demand changes, enabling sustained operational efficiency.

One limitation of this project is that our analysis was based on aggregate demand without incorporating SKU-level variability or seasonality. Additionally, the focus was solely on shipments from DCs to FCs and Stores. While replenishment frequency changes were modeled at the spoke level (FCs and Stores), it was assumed that DC inventory levels would remain constant. Exploring the impact of replenishment frequency changes across all echelons, including DCs, could provide a more comprehensive optimization.

Future work could focus on expanding the model to incorporate SKU-level demand patterns and developing a multi-echelon inventory optimization framework to better capture network-wide change in inventory level. Additionally, the current transportation cost per pound is the quote the sponsor company has for direct shipments only. Given that we are transferring to a transportation network that includes milk runs, we suggest that the sponsor company request a new quote and update the transportation cost per pound to reflect actual costs.

References

- Bakir, I., Erera, A., & Savelsbergh, M. (2021). Motor Carrier Service Network Design. In T. G. Crainic, M. Gendreau, & B. Gendron (Eds.), *Network Design with Applications to Transportation and Logistics* (pp.427-467). Springer.
- Braekers, K. & Ramaekers, K. & Nieuwenhuyse, I. (2015). The Vehicle Routing Problem: State of the Art Classification and Review. *Computers & Industrial Engineering*. 99. 10.1016/j.cie.2015.12.007.
- Chopra, S., & Meindl, P. (2016). *Supply Chain Management: Strategy, Planning, and Operation* (6th ed.). Pearson.
- Desrochers, M., & Laporte, G. (1991). Improvements and extensions to the Miller-Tucker-Zemlin subtour elimination constraints. *Oper. Res. Lett.*, 10, 27-36.
- Rushton, A., Croucher, P., & Baker, P. (2017). *The Handbook of Logistics and Distribution Management* (6th ed.). Kogan Page.
- Vidal, T. (2022). Hybrid genetic search for the CVRP: Open-source implementation and SWAP* neighborhood. *Computers & Operations Research*, 140, 105643.
- Wouda, N. A., Lan, L., & Kool, W. (2024). PyVRP: A high-performance VRP solver package. *INFORMS Journal on Computing*, 36(4), 943–955.
- Zhang, H., Ge, H., Yang, J. et al. Review of Vehicle Routing Problems: Models, Classification and Solving Algorithms. *Arch Computat Methods Eng* 29, 195–221 (2022).
- Zipkin, P. H. (2000). *Foundations of Inventory Management*. McGraw-Hill.