Evaluating Inventory Risk Pooling Strategy for Multi-Echelon Distribution Network

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ABSTRACT

Rising inventory costs is an ongoing challenge for any firm. These costs are of special significance to retail firms like Coppel, whose inventory investments are typically high and margins are slim. Inventory risk pooling is a strategy that is often ignored but can help bring significant cost reduction without affecting service levels. Such inventory decisions are generally considered tactical and are often constrained by the strategic network design decisions that precede them. This siloed approach leads to sub-optimal decisions. The objective of this study is to integrate the two to overcome this inflexibility and evaluate whether the existing distribution network can be profitably reconfigured to introduce risk pooling. This paper develops a decision support model for Coppel to identify the best location for pooling inventory while minimizing total supply chain costs. Integrating inventory and location decisions represents a Location-Inventory Problem (LIP), which comes with inherent modeling and computational complexities due to increased problem size and non-linearity. Evolving solution techniques and improving computing power now make it feasible to solve LIPs efficiently. We develop a Mixed Integer Nonlinear Programming model that follows a Guaranteed Service Model approach to solve this integrated LIP in a multi-echelon multi-product supply chain environment. Due to the non-linear nature of the model, we deploy piecewise approximation methods to first linearize the function before solving. Our research demonstrates that reconfiguring the existing network to introduce risk pooling could reduce the supply chain costs of major product classes by 15%, without affecting their service levels. This is a common challenge across industries. Therefore, the benefits of this research extend beyond Coppel and retail industry.

Capstone Advisor: Dr. Nima Kazemi
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Angelica Bojorquez and Hari Sharma

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- To Angelica, for being an amazing research partner. Her insights and enthusiasm constantly encouraged us to move forward. I couldn’t have asked for a better partner. Her deep knowledge about Coppel supply chain was very valuable.
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Hari Sharma

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1 INTRODUCTION

Inventory is often one of the largest asset investments in the traditional retail industry. Reduction in inventory levels can not only free up much needed cash for the firm but also free up premium warehouse space and related investment costs. Continuous drive towards inventory efficiency can, therefore, give financial strength to a firm and differentiate its performance from the competitors. A survey found that best-in-class retailers had Annual Carrying Cost as Percentage of Revenue of 0.5%, reflecting their inventory efficiencies, whereas an average retailer scored 1.7% (Government of Canada, 2018; Bureau, US Census, 2020; Supply Chain Consortium Survey, 2010). Coppel, our sponsoring company ("company"), performs at 2.7%.

Founded in 1941, Coppel is third largest integrated department stores in Mexico (Euromonitor International, 2020). Coppel has around 1500 stores, 30 Distribution Centers, more than 100,000 employees, and annual revenue of around US$8.0bn. In addition to regular stores, Coppel also has a popular online store, which is ranked among the top 10 retail sites in Mexico (eMarketer, 2019).

Considering the size and complexity of its supply chain, the company believes that there is a significant opportunity and need to optimize its inventory to gain a competitive edge. Per the current Inventory policy at Coppel, inventory is primarily placed at the regional DCs (Distribution Centers) close to the demand point as illustrated in Figure 1-1. Therefore, every regional DC holds a high level of inventory to mitigate the risk of demand uncertainty and meet service level targets. Inventory risk pooling has been a known alternate strategy, where inventory is pooled at a few strategically located DCs and distribution to a downstream location is postponed as long as possible without compromising on service levels (Pagh & Cooper, 1998). Such an inventory postponement strategy has significant potential to optimize the inventory levels across the network and reduce associated costs (Oeser & Romano, 2016).

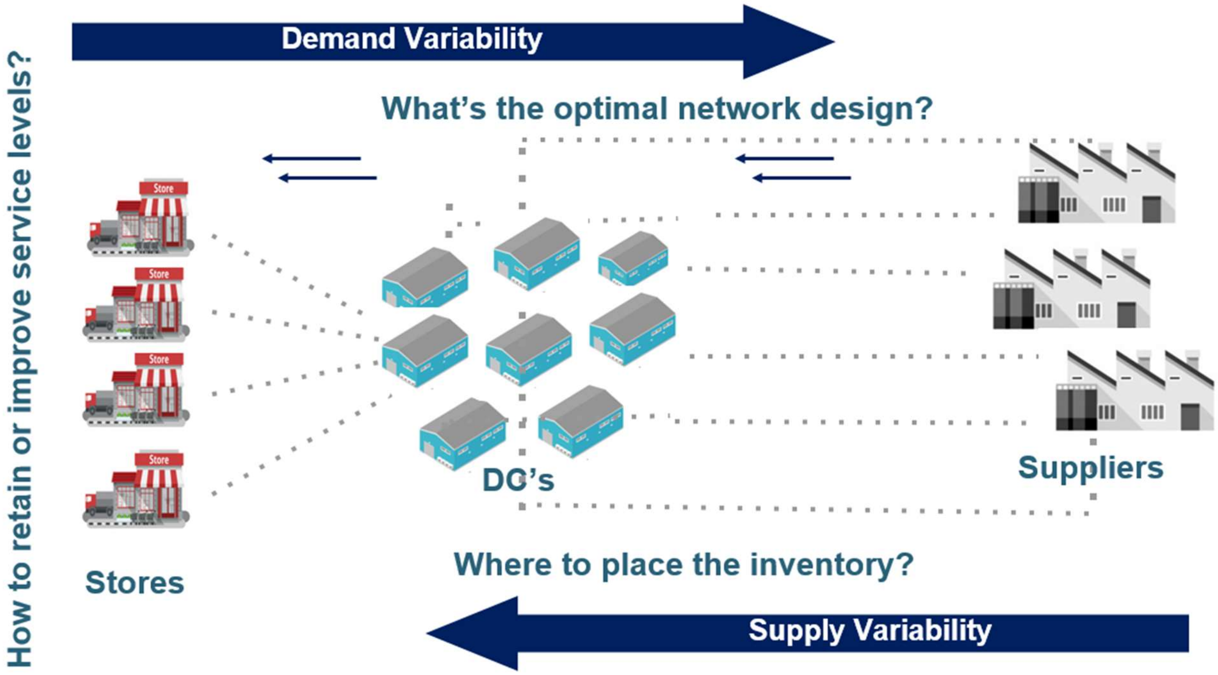

Figure 1-1 Coppel's distribution network representation

Inventory risk pooling strategy presented an excellent opportunity for Coppel to uncover value and gain efficiencies. The company estimated that inventory reduction of 6 days of supply at the DCs by pooling could save around US$18 million in holding cost annually. Additionally, pooling could allow the company to utilize economies of scale to deploy efficient storage techniques resulting in up to 11% improvement in space utilization for their Hardline Products alone. This was equivalent to 1 DC in space, translating into savings of around USD $90M of future capital investments in opening new DCs and around 900 additional headcounts. Risk pooling was also estimated to reduce cross DC inventory movements by 20% annually, eliminating associated costs and lowering its carbon footprint. Further, after consolidation, suppliers would need to deliver in bulk to fewer pooling centers and potentially at lower transportation cost. By accounting for supplier transportation costs, Coppel intended to collaborate and renegotiate prices with suppliers to share the benefits.

Considering these potential benefits for Coppel, the goal of this research was to evaluate our hypothesis that introducing a pooling strategy for Coppel's existing multi-echelon distribution network would reduce inventory and the total operating cost. Simultaneous optimization of the distribution network (Location Problem) and inventory placement (Inventory problem) introduces non-linearity and inherent modeling complexities. We formulated a Mixed-Integer Non-Linear Programming (MINLP) mathematical model in conjunction with Guaranteed Service Model (GSM) approach for this Location Inventory Problem (LIP), which could recommend the best choice of distribution network and inventory placement by minimizing total supply chain cost without compromising on customer service levels. To best of our knowledge, there are no integrated LIP models that would dynamically recommend a network reconfiguration and suggest inventory placement at optimal total supply chain cost while retaining the service levels. Although this research was conducted in context of our sponsoring company, we believe this can be easily adapted to fit the realities of other companies irrespective of industry vertical, geographical location and size.

This document is organized as follows. In Chapter 2, we review relevant research done on risk pooling and modeling techniques available to solve such LIPs. Chapter 3 describes our methodology including data collection, data analysis and mathematical model formulation. The optimization model is presented with detailed description of objective function and constraints applicable in this context. We deploy piecewise linearization technique to reformulate the model for computational efficiency. In Chapter 4, we discuss the results to show how risk pooling or consolidation saved 15% in cost for Coppel's top selling products. This is a significant savings that can improve both financial and operational efficiencies of companies. Managerial insights and conclusions are provided in Chapter 5.

2 LITERATURE REVIEW

We review the research work done in the past to study benefits and challenges of introducing risk pooling in multi-echelon supply chain networks. In addition, we explore different modeling techniques available to concurrently solve both location and inventory problems. We draw on the insights from these research papers to guide us determine the approach best suited to solve the problem in context of Coppel.

Traditionally, researchers have classified supply chain decisions into strategic, tactical and operational decisions, depending on the planning horizon. These decisions are generally taken in hierarchical sequence. Supply chain network design is considered a strategic decision where alternative supply chain structures are evaluated to identify the optimal option to profitably serve customer demand. Decisions include identifying the right facility location, its capacity levels, and a logistics network that connects the various supply chain nodes efficiently and cost-effectively. Strategic decisions typically involve greater capital investments, need a longer time to implement and have longer planning horizons. They lay the foundation for subsequent tactical and operational planning decisions, where the supply chain network is assumed to already exist (Schmidt & Wilhelm, 2000; Baghalian, Rezapour, & Farahani, 2013; Zheng, Yin, & Zhang, 2019).

In contrast, inventory policy decisions are considered tactical in nature where supply chain managers determine inventory levels at each of those supply chain nodes so that customer demand can be served at minimal cost, within the service level targets, and within inventory budget (Schmidt & Wilhelm, 2000). These decisions can have a significant influence on the balance sheet and cash flows, especially in industries like retail that require major investments in inventory. Risk pooling is one such policy decision where inventory is consolidated at a central location instead of multiple downstream supply chain nodes. Eppen (1979) demonstrated how risk pooling can significantly reduce inventory investments.

The potential of deploying such a risk pooling technique in a multi-echelon supply chain is, however, constrained by the supply chain network structure itself, which is assumed to already exist. On the other hand, not considering the implication of pooling on holding costs and capacity requirements during network design can lead to a sub-optimal network. This interdependency makes it imperative to integrate risk pooling decisions within network optimization, especially for industries dealing with supply chain uncertainties and high cost of inventory (You & Grossmann, 2008; Baghalian, Rezapour, & Farahani, 2013; Hiassat, Diabat, & Rahwan, 2017).

Such an integrated approach represents a Location-Inventory problem. LIP comes with inherent modeling and computational complexities due to increased problem size and non-linearity. Though this has been a focus of research for past three decades, there has been significant increase in interest only in recent years (Farahani, Rashidi Bajgan, Fahimnia, & Kaviani, 2015).

We reviewed past work done in this field to determine the most robust and suitable approach to solve Coppel's business case. The company already had a set of DCs, retail stores and supplier sites. As part of our model, we limited the location decision to identify which of these DCs were best suited for pooling and which other DCs they should serve when we introduce risk pooling. The assignment of retail stores to DC is known *a priori* and were not in scope of optimization.

2.1 Inventory Risk Pooling in Multi-echelon Networks

It is necessary to maintain adequate safety stock to address the risks associated with supply chain uncertainties. Inventory risk pooling is the strategy of reducing the variability by aggregating the demand at a central location. This "risk pooling effect" reduces the total safety stock required in the network without increasing the risk of shortage (Kurata, 2014).

Maister (1976) was one of the early researchers to study the effect of pooling inventory at a central location. Maister described the relationship between the number of stocking points and the total inventory in the network with the “Square Root Law of Locations” (SRL). This law affirms that total inventory in the network is proportional to square root of locations where they are stocked, subject to certain conditions. Total inventory includes cycle stock and safety stock. In other words, the total inventory costs in a centralized system (demand met by inventory pooled at a common central warehouse) will be significantly lower than a decentralized system (demand met directly by inventory stocked at individual lowest echelon) if those conditions are met.

Oeser & Romano (2016) note that there is general lack of consensus among researchers regarding these conditions, whether these conditions can be realistically met and which part of inventory SRL applies to. Based on their research and case studies, they provide a list of assumptions where SRL would apply to cycle stock, safety stock, and total inventory. In practice, many companies do not fulfil those assumptions and may find SRL to overestimate the inventory reduction. They also mention that traditionally, companies consolidate safety stock inventory to benefit from the “risk pooling effect”, rather than cycle stock. Centralizing cycle stock will increase transportation cost and delivery times which will lower the net savings.

We illustrate the savings for safety stock with an example. Suppose, there are 4 identical Distribution Centers in the network and receive stock directly from the Supplier as shown in Figure 2-1. Each Distribution Center requires a safety stock quantity *s* to cover for demand with standard deviation σ i.e., total safety stock required is (4**s*). If the firm pools this safety stock at say 1 central Distribution Center instead, then the standard deviation of demand at the central node can be calculated by adding the variability i.e., $\sqrt{(4*\sigma^2)}$ or 2* σ. As safety stock is proportional to standard deviation of demand over lead time, the total safety stock requirement will be reduced to (2**s*) due to “risk pooling effect”, which is a 50% reduction in theory. Though in practice, it will be difficult to meet the condition to achieve that degree of savings, this simplistic example demonstrates the potential of risk pooling. Even a relatively smaller saving can be of special significance to retail industry where firms typically operate on slim margins and inventory is one of the major investments.

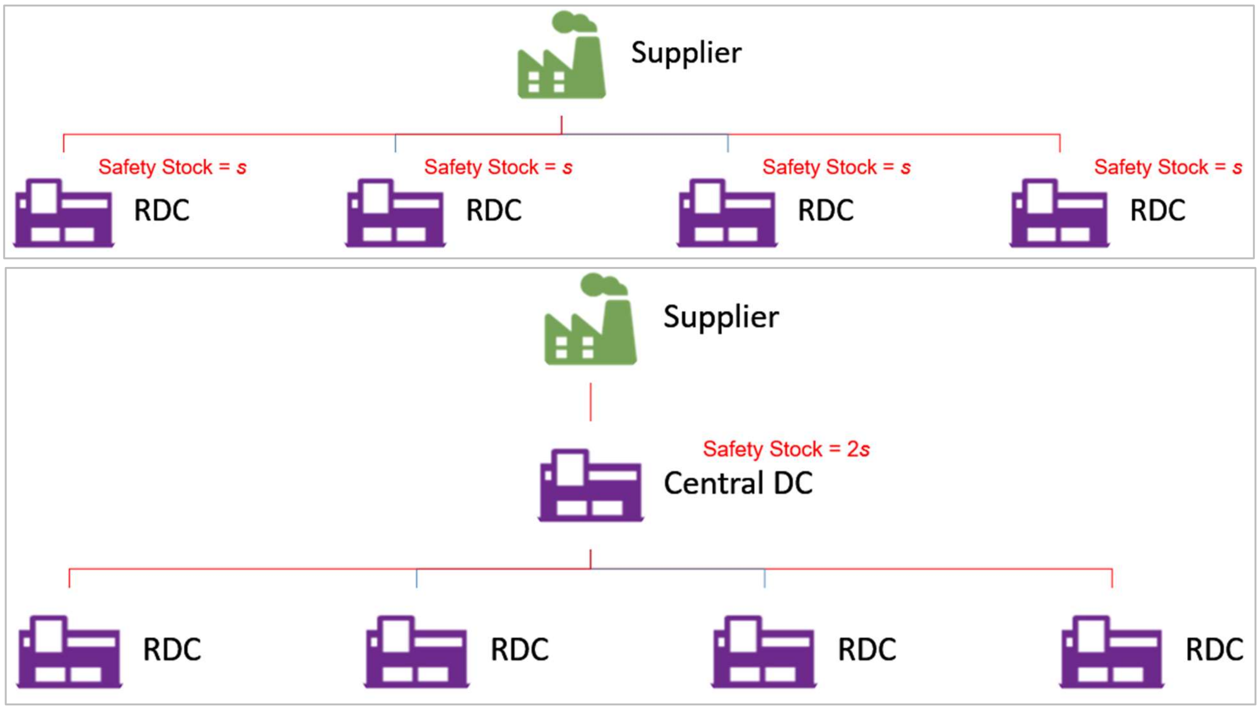

Figure 2-1 Impact of risk pooling on safety stock

Inventory risk pooling decision cannot be taken in isolation without considering its implications on delivery lead time and Service Levels at the lowest echelon. Service Level is an important measure that reflects the probability of firm being able to service customer's demand without stock outs. Research has shown that if a certain minimum Service Levels is maintained at downstream node, pooling can help reduce total inventory cost (Kurata, 2014).

2.2 Location-Inventory Problem

Broadly, Location Inventory Problems can be classified into four types - the Basic LIP, Dynamic-Location Inventory Problem (DLIP), Location-Inventory-Routing Problem (LIRP) and Inventory-

Transportation Problem (ITP). These types of problems focus on strategic decisions such as capacity planning, allocation decisions that, in concert with a location problem can compound supply chain network design problem. (Farahani, Rashidi Bajgan, Fahimnia, & Kaviani, 2015). This study deals with Basic LIP as highlighted in Figure 2-2.

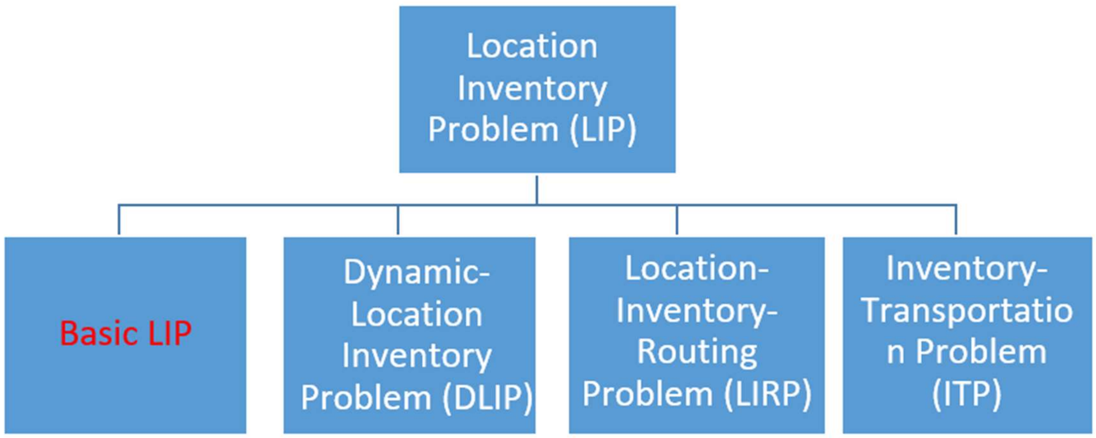

Figure 2-2 LIP model classification

Location-inventory problems in a multi-echelon environment come with inherent complexities due to non-linearity. In addition, these optimization models must handle a large number of products, supply chain nodes, constraints and scenarios. All this significantly increases the complexity and size of the model requiring sophisticated methods to solve. Evolving techniques and continuously improving computational capabilities allow us to deal with such large and complex problem now than ever before (Farahani, Rashidi Bajgan, Fahimnia, & Kaviani, 2015; Fahimnia, Farahani, Marian, & Luong, 2013; Zheng, Yin, & Zhang, 2019).

Few such key challenges in the context of the capstone project are discussed below.

1 **Non-linearity**: Location Inventory problems are non-linear by nature. An appropriate approximation technique had to be identified to convert the MINLP problem into one that could be solved through simpler MILP techniques. We discuss this in Section 2.3

2 **Lead-time Stochasticity**: Replenishment lead time in a single echelon network is exogenous to the system and can generally be treated as a known input to the model. In contrast, a multi-echelon inventory system involves multiple nodes where inventory may be stored, and inventory flows from one node to another. The replenishment time or service time required by an upstream node to fulfill a demand from a downstream node will depend on its stock availability. When the upstream node is unable to fulfill from its stock, the downstream node will experience a delay. This makes the inter-nodal lead times stochastic even when the order processing time for the supplying node could be deterministic. (Graves & Willems, 2003). Non-deterministic nature of inter-nodal service time variability had to be appropriately accounted for within the model. We discuss this in Section 2.3

3 **Data Volume**: Coppel's enormous network of DCs and large product portfolio could make the model complex and computationally challenging. An appropriate level of data aggregation and the right scope to focus had to be determined. We discuss this in Chapter 3

2.3 Solution Techniques

Non-linearity: Researchers have explored various exact, heuristic and metaheuristic algorithms to solve LIP models. Exact methods guarantee optimality but their run time often increases dramatically with problem size. Exact methods that have been used for LIP problem include Branch & Bound (B&B), Column Generation, Dynamic Programming, and Generalized Benders' Decomposition. Among these, B&B is commonly used. On the other hand, heuristic methods can solve problems more quickly but do not guarantee optimality. But they are often useful when exact methods are too slow or fail to find exact solution. Heuristics methods that have been used for LIP problem includes greedy algorithms, dropping, adding, substitution and exchange of elements of model. Additionally, metaheuristic techniques that have been used for LIP problem includes Lagrangian relaxation, piecewise linear approximation, genetic algorithm, simulated annealing and tabu search (Farahani, Rashidi Bajgan, Fahimnia, & Kaviani, 2015).

Lagrangian Relaxation technique is especially considered very useful in simplifying these optimization problems by removing “bad” constraints (hard to solve) and adding them into the objective function instead. A penalty is applied whenever the constraint is violated (Fisher, 1981).

Choosing piecewise linear approximation technique for being more precise approximation methods than Lagrangian relaxation, and applied as You & Grossman (2010) did it.

Piecewise linear approximation: is used to approximate different types of nonconvex nonlinear functions, especially for univariate concave functions and square root term by set of intervals, the more number of intervals, the better is the approximation of the nonlinear function, but more additional variables and constraints are required. Using the multiple choice formulation to approximate the univariate concave term and the objective function of the problems, match better approximation.

Lead time Stochasticity: Two main approaches have been established to model time delays in material flow within multi-echelon supply chains: Stochastic-Service Model and Guaranteed-Service Model (Graves & Willems, 2003; Sena et al, 2015; You & Grossmann, 2008). Both are built as periodic review policies and both consider stochastic demand. The main difference between them is that the Stochastic-Service Model (SSM) approach tries to minimize the total expected holding and backordering cost by optimizing the target inventory level at each stage. In contrast, Guaranteed-Service Model (GSM) approach minimizes the total safety stock cost by optimizing the service time between nodes. The consequence of these formulations, as Simchi-Levi and Zhao (2012) state, is that the SSM will have a stochastic service time between nodes as there will be delays when a stock-out event happens, while GSM will have a deterministic service time where peaks in demand are dealt with extraordinary measures. For detailed discussions and comparisons on these two models, refer to Graves & Willems (2003) and (Eruguz et al., 2016).

In this paper, we choose the Guaranteed Service Model approach because of its computational efficiency (Eruguz et al., 2016). Hence, we find it pertinent to briefly describe GSM approach and also highlight optimization models other researchers have used in conjunction with Guaranteed-Service Model.

Guaranteed Service Model: In this approach, each node *j* in a multi-echelon distribution system offers a guaranteed service time, say T_j. In other words, if a node *j* receives a demand from its downstream node at time t, it promises to make the material available and ready to ship by time $t + T_j$. Additionally, each node has a deterministic order processing time γ $(gamma)$, and is the time gap between when demand is received and when it is available to be served as shown in Figure 2-3. This time period includes material handling time, transportation time and review period (You & Grossman, 2008).

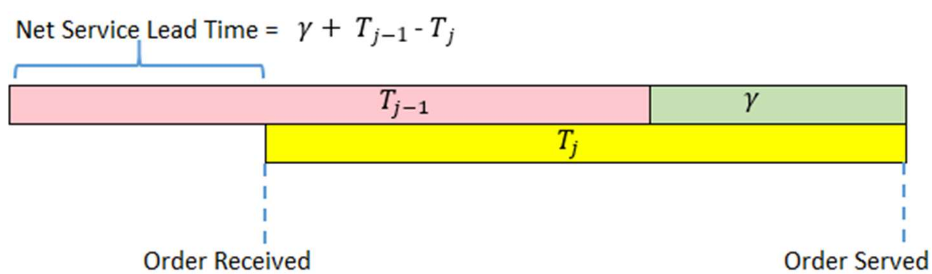

Figure 2-3 Relationship between various time components in Guaranteed Service Model

Therefore, the net service lead time, which is the duration node *j* must manage demand variations with its safety stock, can be defined in terms of guaranteed service time of predecessor T_{j-1}, the order processing time γ, and the guaranteed service time to its successor T_j as shown in Figure 2-3. This net services time is a decision variable to be optimized. On the other hand, guaranteed service time for the terminal nodes serving the customer demand is deterministic and an exogenous input.

Shen et al. (2003) developed a Location Allocation Risk-Pooling Model where the Distribution Centers order inventory from the plant using the Economic Order Quantity model. They define an integrated model using Column Generation technique to optimize Distribution Center locations, safety stock level at each DC, and identifying which DCs should supply to which retailer store. As the lead time is considered constant and deterministic, we can state that this follows the philosophy of the Guaranteed Service Model approach. Gupta (2008) describes a case study where he develops a methodology to quantitatively make supply chain design decisions and inventory policies decision. He does this sequentially. For a given set of warehouses in an upstream level and a set of distribution centers in a downstream level, he first determines which warehouse must serve which distribution center in order to minimize transportation cost, safety stock holding cost and pipeline inventory cost. Later, he uses a programming model based on the Guaranteed Service Model approach to determine the safety stock placement for each node. Miranda and Garrido (2009) proposed a two-step formulation across a sequential heuristic approach optimizing service level, inventory decisions and network design decisions simultaneously. During the first step, it optimizes service level and the second step addresses location and inventory decisions. They use algorithm iterations to solve the location inventory model for a fixed service level and recalculate service level to find a balance between the operating system and unfulfilled demand cost.

You and Grossman (2010) developed a joint multi-echelon supply chain design and inventory management model for defining both kinds of decisions at the same time. The base for the development of this model is the Guaranteed Service Model, as they define what is going to be the guaranteed service time for each node. By the nature of this problem, the defined model is a Mixed Integer Non-Linear Programming Model, which is relaxed to a linear problem using approximations such as Lagrangian relaxation (LR) and piece-wise linear approximation. Harahap et al (2019) also used the Guaranteed Service Model approach to model the network to try to optimize facilities location, production, inventory, and transportation decisions. In this case, the decision variables involve opening warehouses, which warehouses are going to serve which retailers, which routes are going to be used, how many products are

Author	Type of Model	Horizon	Inventory Policy	Plant/Supplier Capacity	DC Capacity	Number of DCs	Demand Type	Shipping Cost	Product	Order size capacity	Facility Installation Cost	Facility Operating Cost	Inventory Holding Cost	Ordering Cost	Safety stock cost	Service level	Shipment cost	Transportation cost	
(Shen et al., 2003)	LIP	I	F	I	I	Ex	D	OF	S	F	●			●	●		●	●	
(Ozsen et al., 2008)	LIP	I	C	I	F	En	S	OF	S	F	●		●	●	●				
You & Grossmann (2008)	LIP	I	P	I	I	Ex	S	OF	S	I	●	●	●		●	●		●	LR &
(Diabat et al., 2009)	LIP	I	C	I	I	En	D	OF	S	F	●		●	●	●	●	●	●	
(Jeet et al., 2009)	LIP	I	C	F	F	En	S	D&OF	S	I	●	●	●		●	●		●	
(Liao & Hsieh, 2009)	LIP	I	C	F	F	Ex	S	OF	S	F	●	●			●	●		●	
(Miranda & Garrido, 2009)	LIP	I	P		F	Ex	S	OF	S	F	●		●	●	●	●		●	
Yao et al. (2010)	LIP	I	P	F	F	En	S	S&OF	M	F					●	●		●	
(Nasiri et al., 2010)	LIP	I	C	F	F	Ex	S	D&OF	M	F	●		●		●				
Schuster Puga & Tancrez (2017)	LIP	I	C	I	I	En		DF	S	F	●		●	●	●	●		●	
Our Work	LIP	I	P	I	F	Ex	S	OF	M	F			●	●	●	●		●	Pi

going to be produced and delivered. This is also a Mixed Integer Programming model which is solved through a direct search algorithm.

2.4 Summary

In this chapter, we reviewed the studies close in scope to our research.

Table 2-1 compares our capstone with the past studies and highlights the similarities and differences between our study and the ones in the literature. Based on the assessment, presented in Table 2-1, we deduce that the scope of research done by You & Grossmann (2008) is particularly relevant to our work. However, there are several differences between our work and You & Grossmann (2008) for which we had to adapt. Differences in model characteristics are shown in

Author	Type of Model	Horizon	Inventory Policy	Plant/Supplier Capacity	DC Capacity	Number of DCs	Demand Type	Shipping Cost	Product	Order size capacity	Facility Installation Cost	Facility Operating Cost	Inventory Holding Cost	Ordering Cost	Safety stock cost	Service level	Shipment cost	Transportation cost	
(Shen et al., 2003)	LIP	I	F	I	I	Ex	D	OF	S	F	●			●	●		●	●	
(Ozsen et al., 2008)	LIP	I	C	I	F	En	S	OF	S	F	●		●	●	●				
You & Grossmann (2008)	LIP	I	P	I	I	Ex	S	OF	S	I	●	●	●		●	●		●	LR &
(Diabat et al., 2009)	LIP	I	C	I	I	En	D	OF	S	F	●		●	●	●	●	●	●	
(Jeet et al., 2009)	LIP	I	C	F	F	En	S	D&OF	S	I	●	●	●		●	●		●	
(Liao & Hsieh, 2009)	LIP	I	C	F	F	Ex	S	OF	S	F	●	●			●	●		●	
(Miranda & Garrido, 2009)	LIP	I	P		F	Ex	S	OF	S	F	●		●	●	●	●		●	
Yao et al. (2010)	LIP	I	P	F	F	En	S	S&OF	M	F					●	●		●	
(Nasiri et al., 2010)	LIP	I	C	F	F	Ex	S	D&OF	M	F	●		●		●				
Schuster Puga & Tancrez (2017)	LIP	I	C	I	I	En		DF	S	F	●		●	●	●	●		●	
Our Work	LIP	I	P	I	F	Ex	S	OF	M	F			●	●	●	●		●	Pi

Table 2-1. You & Grossmann (2008), like other studies we reviewed, also assume that the set of facilities in each tier of the network is pre-determined and the sets are mutually exclusive. In contrast, we have a common set of facilities, and the model has to dynamically determine which of those facilities should consolidate the inventory and which other facilities they should serve. Unlike You & Grossmann (2008), our model can handle multiple products. This flexibility makes it more convenient and realistic for use in practice by planners.

Like You & Grossmann (2008), we adopt Guaranteed Service Model to handle service time stochasticity. Coppel's transportation policies require a fixed schedule for shipment operations between DCs and stores. Considering Coppel's transportation policy and the stochastic nature of the lead times, using a Guaranteed Service Model will allow us to better model the time delays between the supply chain nodes in this Capstone project. Guaranteed Service time will be a parameter that will be known for each node. We assume that the demand for the product categories in scope is stochastic but bounded at a certain level.

Based on our research, we use Piecewise Approximation techniques to simplify and solve the MINLP problem.

Author	Type of Model	Horizon	Inventory Policy	Plant/Supplier Capacity	DC Capacity	Number of DCs	Demand Type	Shipping Cost	Product	Order size capacity	Facility Installation Cost	Facility Operating Cost	Inventory Holding Cost	Ordering Cost	Safety stock cost	Service level	Shipment cost	Transportation cost	Solution Technique
(Shen et al., 2003)	LIP	I	F	I	I	Ex	D	OF	S	F	●			●	●		●	●	Column Generation
(Ozsen et al., 2008)	LIP	I	C	I	F	En	S	OF	S	F	●		●	●	●				Heuristics
You & Grossmann (2008)	LIP	I	P	I	I	Ex	S	OF	S	I	●	●	●		●	●		●	LR & Piecewise linear approximation
(Diabat et al., 2009)	LIP	I	C	I	I	En	D	OF	S	F	●		●	●	●	●	●	●	Heuristics
(Jeet et al., 2009)	LIP	I	C	F	F	En	S	D&OF	S	I	●	●	●		●	●		●	Outer Approx
(Liao & Hsieh, 2009)	LIP	I	C	F	F	Ex	S	OF	S	F	●	●			●	●		●	Hybrid Genetic Algorithm
(Miranda & Garrido, 2009)	LIP	I	P		F	Ex	S	OF	S	F	●		●	●	●	●		●	Sequential Heuristics
Yao et al. (2010)	LIP	I	P	F	F	En	S	S&OF	M	F					●	●		●	Iterative Heuristics
(Nasiri et al., 2010)	LIP	I	C	F	F	Ex	S	D&OF	M	F	●		●		●				LR & Sub-gradient
Schuster Puga & Tancrez (2017)	LIP	I	C	I	I	En		DF	S	F	●		●	●	●	●		●	Iterative Heuristics
Our Work	LIP	I	P	I	F	Ex	S	OF	M	F			●	●	●	●		●	Piecewise linear approximation

Table 2-1 Comparison of LIP model characteristics and solution techniques with our work

Horizon	I=Infinite F=Finite	**Product**	S=Single M=Multiple	**Shipping Cost**	DF=Function of Distance OF=Function of Order Size
Inventory Policy	C=Continuous P=Periodic	**Capacity**	I=Infinite F=Finite		D&OF=Function of Distance and Order Size S&OF=Function of Shipment and Order Size
Number of DCs	Ex=Exogenous En=Endogenous	**Demand Type**	S=Stochastic D=Deterministic		

3 METHODOLOGY

This chapter discusses the methodology we adopted to achieve our goal. A high-level overview of the methodology is presented in Figure 3-1, which lists the main activities undertaken during each stage of the project. Remaining part of this document is organized in accordance with these stages.

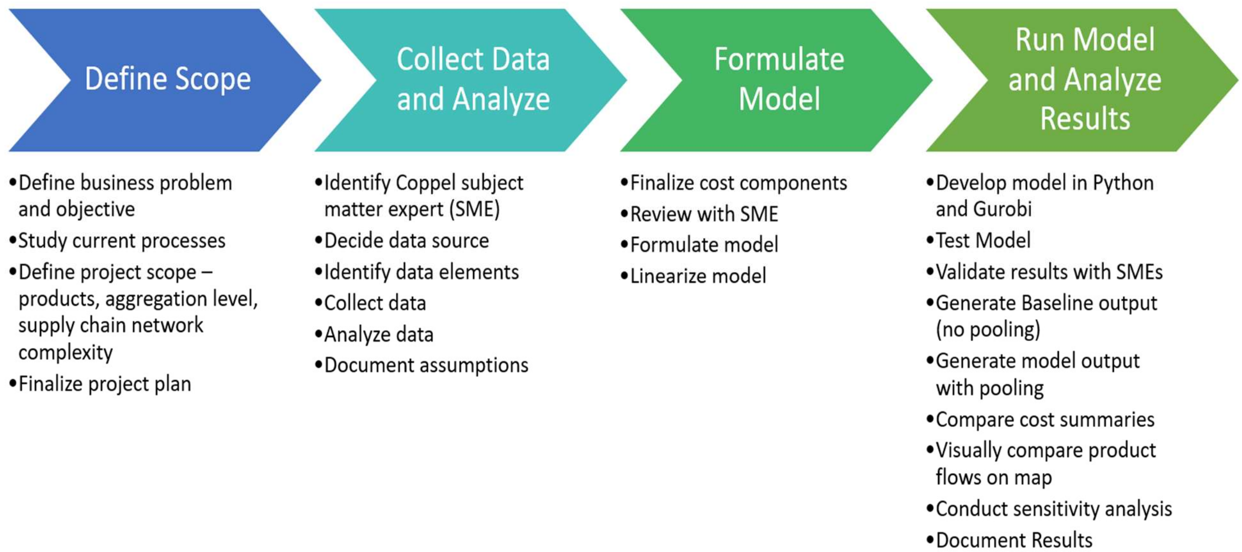

Figure 3-1 Methodology

In Section 3.1, we describe Coppel's processes relevant to this project and rationale for defining the project scope. In Section 3.2, we discuss data collection process, list the relevant data elements we need for modeling and state the associated assumptions we had to make. Section 3.3 details the mathematical model formulation approach, and how we later linearized it. In chapter 4, we present the output from the model and analyze the results.

3.1 Project Scope Definition

Coppel has around 100,000 SKUs that it sells through its network of 1500 physical stores and online channel. This product assortment is supported by around 23 Regional distribution centers, 150 cross dock centers, 700 Full Truck Loads and 1500 small trucks, making it one of the largest supply chain networks within Latin America as seen in Figure 3-2 Coppel's distribution network in MexicoFigure 3-2. The size and variations within Coppel's supply chain presented complexity that had to be carefully studied and the project scope had to be clearly demarcated without compromising on the core objective.

Figure 3-2 Coppel's distribution network in Mexico

We scheduled multiple video conferences to interview Coppel's subject matter experts (SME) from Supply Chain and Information Technology organizations to have a better understanding of Coppel's supply chain processes and associated intricacies. In this sub-section, we briefly describe our assessment of complexities in terms of products and supply chain network, and how that guided the project team and Coppel to jointly agree on the project scope.

3.1.1 Supply Chain Network Complexity

Coppel offered two distinct sales channels to interface with the market – online store and company owned brick-and-mortar stores. Every store had a dedicated distribution center in the region (RDC) assigned to it, which regularly replenished stock at the store. Online orders are fulfilled directly by the RDCs.

These RDCs had two distinct sourcing paths depending on where the product was sourced from:

- Domestically sourced products were received directly from the supplier, and inventory was held for eventual distribution to retail stores within that RDC's region as shown in Figure 3-3
- Internationally sourced products were first received at one of the few Import Distribution Centers near the ports. Import DCs act as cross-docking facilities for shipments to RDCs and eventually further downstream to retail stores within that RDC's region as shown in Figure 3-4

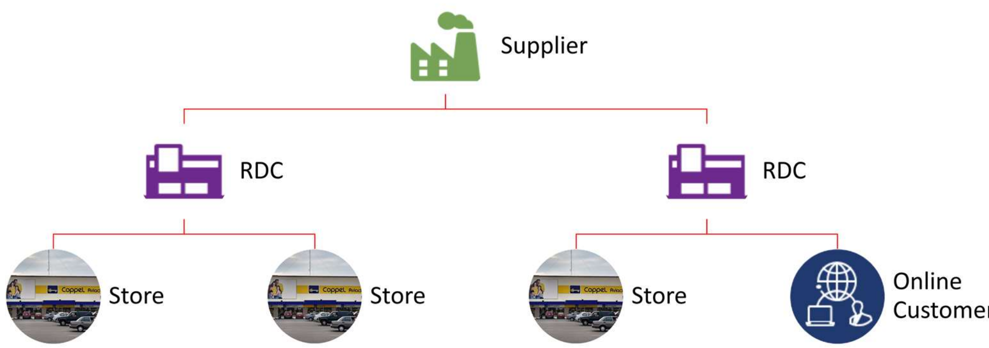

Figure 3-3 Distribution: Domestically sourced products

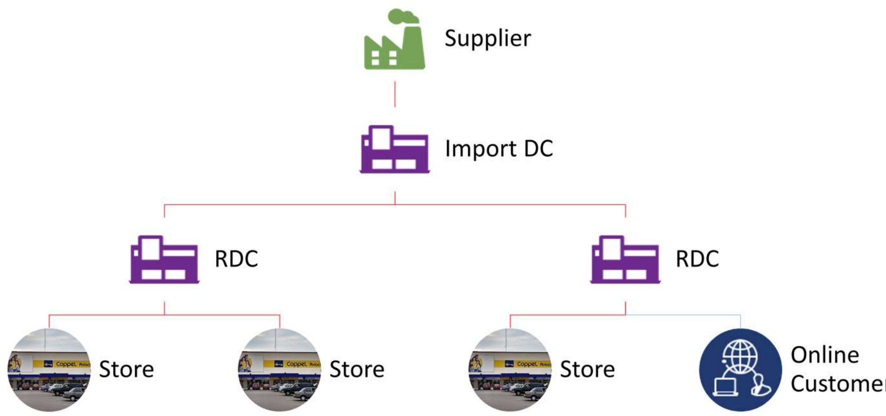

Figure 3-4 Distribution: Internationally sourced products

Table 3-1 summarizes the effective number of internal supply chain echelons that were in play depending both on how product was sourced as well as how it was sold. For instance, products sourced from a domestic supplier and sold through a store would flow internally through the RDC and the store (2 echelons), but product sold online is shipped to customer directly from RDC (1 echelon). Almost all the literature on this topic that we reviewed assume a fixed number of echelons. Our mathematical model had to account for this additional complexity of varying number of echelons.

DEMAND / SUPPLY	ONLINE	STORE
DOMESTIC	1	2
INTERNATIONAL	2	3

Table 3-1 Internal supply chain echelons

Additional insights from the Coppel SMEs revealed few facts that helped reduce this complexity to a certain extent:

- Retail stores did not carry any safety stock as per their supply chain policy
- Company did not intend to change the current RDC-store assignments

- Company did not intend to change the existing supplier-receiving port assignments for internationally sourced products
- Company did not intend to open a new DC or close an existing one

Consequently, the retail stores were excluded from the model as they did not influence the cost function. In case of internationally sourced products, we treat the Import Distribution Centers as fixed external supplying sites. Moreover, the mathematical model has to evaluate the possibility of using one or more of the existing DCs to act as a consolidation or pooling location instead of recommending opening of a new DC.

3.1.2 Product Complexity

Competition and ever changing consumer tastes forces retail firms to continuously offer new products. It was not surprising to find a portfolio of 100,000 SKUs for a large retail organization like Coppel. Our initial Pareto analysis (see Figure 3-5) indicated that a very percentage of products contributed disproportionately to most of the annual holding cost spend. It was essential to focus only on the biggest contributors. Furthermore, for reasons explained in Chapter 2, including these many SKUs would considerably increase the model size. Solving such large LIPs at this level of granularity can quickly become computationally challenging and more importantly, the output less comprehensible for the planners. We, therefore, determine the relevant subset of data we should consider in scope for our assessment and their level of aggregation.

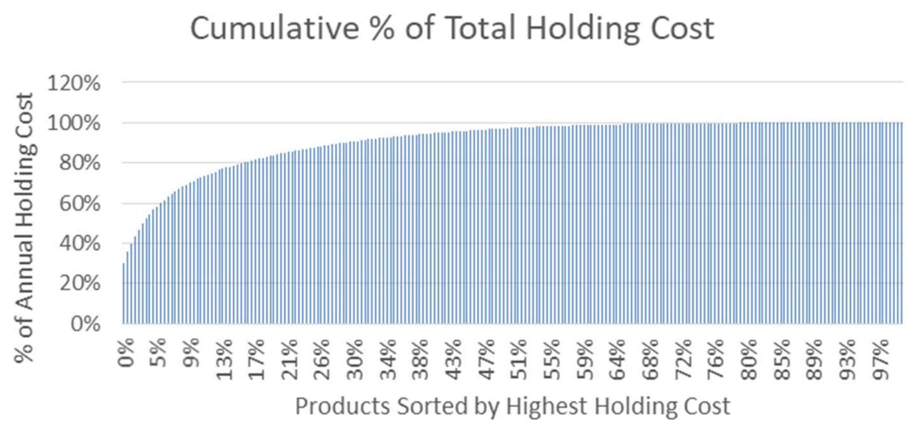

Figure 3-5 Share of annual holding cost by products

Product Aggregation: Product class was one of the most widely used product aggregation level across Coppel. Using product class for aggregation would make it easier for planners to communicate the results with senior management and other departments. Our analysis also showed that SKUs within a given Product Class and that were sourced from a given supplier shared similar logistics characteristics e.g., routes, service level requirements, relative size, shipping and handling methods. Therefore, we decided to aggregate the products by Supplier and Product Class (e.g., Samsung-Refrigerators).

Product Scope: As explained earlier, it was important to limit the focus on a significant few product classes. The decision on number of product classes we could consider was also limited by timely availability of clean data at the level of granularity and effort required to clean it. As the objective of our project was only to evaluate the benefits of introducing risk pooling for Coppel, we decided to focus on top three Product Classes that constituted the highest share of both total annual holding cost and revenue for the company. Reducing the total holding cost for these few key Product Classes could help us demonstrate the potential of introducing pooling strategy to the sponsoring company and at the same time keep the model size manageable. Therefore, we plotted the performance of product classes and compared their contribution to total revenue and total holding cost in Figure 3-6. Each product class is represented by a separate cell. The three major product classes are highlighted in top right quadrant. We focus our attention to these three. Table 3-2 lists their service levels, their contribution to total revenue and holding cost in tabular form. We limited the scope of our data collection to these three classes.

DEPARTMENT	CLASS	% TOTAL HOLDING COST	% TOTAL REVENUE	SERVICE LEVEL (DC)	SOURCED DOMESTICALLY?
ELECTRONICS	COLOR TELEVISION	6%	6%	96%	Yes
WHITE LINES	REFRIGERATORS	4%	5%	83%	Yes
WHITE LINES	WASHING MACHINES	3%	3%	94%	Yes

Table 3-2 Share of holding cost by product classes

Figure 3-6 Performance by product class highlighting top 3 products by revenue and cost

3.2 Data Collection

"Legal System", which is Coppel's custom-built Enterprise Resource Planning (ERP) system, was the primary source of information for our Capstone project. We focused on two process areas for the data required for this Capstone project: “Procurement Control” and “Inventory control”. Coppel appointed two subject matter experts from their planning and logistics organization who constantly supported our project and were primary points of contact. They internally coordinated with other Coppel teams (like Information Technology) to provide any additional information we needed. As the requested data was generally aggregated and files were not large, it was convenient to exchange the data in prescribed Excel format over emails. All data analyses were done using Microsoft Excel features.

We received historical sales data from 2019 to study the product performance and biggest contributors to the inventory costs. This data was aggregated by product class and included information such as total revenue, holding cost, total inventory cost, average inventory at store, average inventory at RDC, fill rate, inventory turns, and days of supply for each of the 214 product classes. We ran summary statistics and analysis, results from which were presented in Section 3.1.2.

We then needed data at more granular level for optimization. As explained in Section 3.1, we needed this data for the three select product classes, aggregated by product class, supplier and RDC. The data elements aligned with those listed in Table 3-3. Additionally, we received separate files for Inter DC transportation costs and one for Inter DC lead times.

We list some of the key data elements required for our model later in this section. We also discuss some of the challenges we faced and logical assumptions we had to make to address them.

3.2.1 Data Elements

As we mentioned earlier, we provide a set of DCs to the model as input. The model recommends a subset of those DCs that should pool the inventory for rest of the DCs. We call such DCs as Consolidation Distribution Centers (CDC) and the remaining DCs as Regional Distribution Centers to distinguish between the two. It is also worth highlighting that this reference will be in context of a specific product class-supplier combination. In other words, a DC may act as a Consolidation Distribution Center for a certain product class-supplier combination but may act as a Regional Distribution Center for a different combination.

We group the data we need for the model into three categories: logistics operations cost, stochastic demand behavior, and supplier procurement process. The data elements to be input in the proposed model are listed in Table 3-3.

Data Category	Data Element	Description
Logistics Operations Costs	Handling cost for CDC	Storage and handling cost of a consolidator DC
	Handling cost for RDC	Storage and handling cost of a non-consolidator DC
	Volume Per Unit	Average volume per unit by product class-supplier-DC (m3/unit). Used to compute transportation and storage cost.
	Inventory Holding Cost	Sum of Storage & Handling Expenses plus Capital Cost by product class-supplier-DC ($).
	Inter DC Lead time	Lead time between two DCs (days).
	Inter DC Transportation cost	Handling and transportation cost caused by reallocation of merchandise ($/m3). Volume is involved in this quantity as Coppel manages this cost is terms of volume.
Stochastic Demand	Annual Demand	Average annual demand by product class -supplier-DC (units)
	Standard Deviation of Demand	Annual demand variability compare to forecast by product class -supplier-DC (units)
Supplier Procurement	Supplier Lead Time	Average lead time from supplier to DC by product class -supplier-DC (days)
	Supplier Transportation Cost	Transportation cost per unit from supplier facility to DC by product class -supplier-DC ($/unit)

Table 3-3 Key data elements

Pooling would allow the Consolidation DC to deploy economies of scale by making better use of space, there is a clear opportunity of 30% of space better utilization. Therefore, the storage and handling costs per unit volume for a DC serving as consolidation point will be lower compared to a DC that does not.

3.2.2 Data Preparation

Data was extracted from the databases mentioned previously in a prescribed Excel template. Level of aggregation and data elements are listed in Table 3-3. There were few inconsistencies between the multiple files received and required few detailed discussions to resolve. One such issue was the use of numeric DC number in the main demand data file, and an alphabetic code to represent DC in the Inter DC lead times and transportation costs file. This was because the files were extracted from different systems. Coppel later provided another file that helped us cross reference the numeric DC code with alphabetic code in two files.

3.2.3 Assumptions

The main assumptions followed by the model formulation are established mainly for the company´s operational model, data behavior and others to meet Guaranteed Service Approach, as the below list:

- Demand is normally distributed: demand over any time interval is also assumed to be normally distributed and bounded (You & Grossman, 2010).
- All distribution centers considered in this scenario can be used as consolidators if the optimal scenario requires them to operate as such. This is assumption is made as all DCs in Coppel Network can operate either as CDC or as RDC.
- Service time between nodes are guaranteed to be deterministic, so we can meet the assumptions required by the Guaranteed Service Model. We also assume that the Supplier Delivery Lead Time is also deterministic. As refer to Graves & Willems (2003), the Guaranteed-Service Model considers service times between tiers and also suppliers Delivery Time as deterministic.
- As discussed by You & Grossman (2010), one critical assumption that the General Service Approach makes is that the demand is bounded by the service level or by the safety stock factor that the manager implements within the model. Any excess of demand will be dealt

with extraordinary measures, such as rescheduling or expediting orders. Therefore, shortage cost is not present within the objective function.

3.3 Model Formulation

To solve the problem presented previously, we develop a MINLP model to solve the multi-echelon LIP by adopting similar work done by Farahani (2015) and You & Grossman (2010). We have made several changes to align to company´s business scenario, having the flexibility to choose any DC´s to be in the first or second tier, while Farahani and You & Grossman models have a fix tier design. Our model formulation assumes a periodic review policy to reflect sponsoring company's inventory policy.

3.3.1 Cost Structure

The objective of the mathematical model is to minimize total supply chain cost. Following cost components have been included in the objective function shown in equation (1):

1) **Facility or Location Cost**: In the proposed model the facilities are always open. Therefore, the fixed facility cost is not considered. However, the facility operating cost of a DC used for consolidation is lower than that of an RDC due to better space utilization. This cost is represented by first two terms in the objective function – one for DCs recommended for consolidation and other for remaining DCs.
2) **Transportation Cost**: We consider transportation cost for shipments from supplier to DC as well as inter-DC transportation cost. Transportation cost is a function of quantity of SKU shipped, and its volume in cube meters per kilometer. These are represented by third and fourth term in the objective function.
3) **Cycle Stock Cost**: Cost of holding the cycle stock at DC is considered in the time we need to wait to receive merchandise from supplier or DC. It is a function of quantity of inventory carried from one period to another considering processing time. This is represented by 5th and 6th term in the objective function.

4) **Safety Stock Cost**: This is the cost of holding buffer inventory to meet certain minimum service level targets considering uncertain nature of demand. Demand is assumed to be normally distributed and z_{α} is the critical value corresponding to the service level targets. Net Lead time determines how much safety stock we need in a DC. These are represented by last two terms in the objective function.

5) **Ordering Cost**: Ordering cost is not considered by the model because company uses periodic review policy with base stock, where the ordering cost as well as ordering frequency is set, and is not something we can optimize in our model.

3.3.2 **Objective Function**

The problem is to determine how many distribution centers (DCs) should consolidate merchandise to others DCs, identifying which others and how long should each quote its service time. A second objective is to define what safety stock inventory level according to the definition if the Guaranteed Service Model.

In this section, we first present the base MINLP mathematical model to solve the LIP. Later, we reformulate the model in order to linearize it using Piecewise Approximation technique. Once the model is linearized, we can use it to find a feasible and optimal solution. Notations used in the model are described in Table 3-4.

Indices	Description
k	Index for $k \in K$, where K is the set of suppliers
i	Index for DC $i \in I$, where I corresponds to the set of all DCs in the second tier
j	Index for DC $j \in J \subseteq I$, where J is the set of DCs in the first tier.
Parameter	
H_j	Inventory holding cost at DC j (\$/units/year).
H_i	Inventory holding cost at DC i (\$/units/year).
F_j	Variable cost for operating DC j where DC j consolidates merchandise ((\$/units/year).
g_j	Variable cost for operating DC j where DC j does not consolidate merchandise ((\$/units/year).
l_{kj}	Mean of delivery lead time from supplier k to DC j (days)
μ_{ki}	Mean of demand of merchandise shipped from supplier k to DC i. This parameter aggregates the demand of all clients whose products are shipped from DC i. (days)
σ_{ki}^2	Variance of demand, in days, of merchandise shipped from supplier k to DC j. This parameter aggregates the demand of all clients whose products are shipped from DC i (units^2).
w_{kj}	Transportation cost from supplier k to DC j (\$/units)
t_{ji}	Transportation cost per unit from DC j to DC i (\$/m^3).
z_α	Critical value from the standard normal distribution, which corresponds to the $(1-\alpha)$th quantile.
ψ	Number of working days per year (days)
λ	Service delivery time from DC j or DC i to stores (days)
γ	Processing time per unit within a DC (days)
$LT_{kj}Max$	Maximum Lead Time at which each supplier can ship merchandise to DC j
n_{ji}	Lead time between DC j to DC i.
$F1_{kj,p_1}$	For the linearized segment p_1 for supplier k and DC j, represents the intercept term of the linear equation.
$C1_{kj,p_1}$	For the linearized segment p_1 for supplier k and DC j, represents the slope of the linear equation.
$F2_{ki,p_2}$	For the linearized segment p_2 for supplier k and DC i, represents the intercept term of the linear equation.
$C2_{ki,p_2}$	For the linearized segment p_2 for supplier k and DC i, represents the slope of the linear equation.
Binary variables	
X_{kj}	Binary variable: equals 1 if DC j is going to receive shipments from supplier$k$; 0 otherwise.
Y_{kji}	Binary variable: equals 1 if DC j is going to ship merchandise from supplier k (that this DC j already received) to DC i.
Z_{kj}	Binary variable: equals 1 if DC j is going to enter a consolidation scheme. In other words, if DC j is going to send shipments to other DCs.
Continuous variable	
N_{kj}	Positive real-valued variable; determines the Net Lead Time at which DC j is going to ship new orders to the next tier in the multi-echelon network for merchandise from supplier k. Determines how much safety stock do we need in DC j.

L_{ki}	Positive real-valued variable; determines the Net Lead Time at which DC i is going to ship new orders to the customers for merchandise from supplier k. Determines how much safety stock do we need at DC i, which receives inventory from the consolidator.
S_{kj}	Positive real-valued variable; determines the guaranteed service time that DC j requires to process and send a new order until it arrives at the next node.
Auxiliary variables	
YZ_{kji}	Continuous non-negative variable as a product of the decision variables Z_{kj} and Y_{kji}.
SY_{kji}	Continuous non-negative variable that represents the product of S_{kj} and Y_{kji}.
$SY1_{kji}$	Continuous non-negative variable that holds the value of S_{kj} when the product of S_{kj} and Y_{kji} results in zero.
NY_{kji}	Continuous non-negative variable that represents the product of N_{kj} and Y_{kji}.
$NY1_{kji}$	Continuous non-negative variable that holds the value of N_{kj} when the product of N_{kj} and Y_{kji} results in zero.
NYV_{kj}	NYV_{kj} Is a linear combination of N_{kj} where the coefficients are the sum of the variances of the daily demand for each DC i supplied by DC j.
NYV'_{kj,p_1}	Decision variable that takes the value of NYV_{kj} if that value is contained in the linearized segment p_1.
XZ_{kj}	Continuous non-negative variable as a product of the decision variables X_{kj} and Z_{kj}.
XYZ_{kjj}	Continuous non-negative variable as a product of the decision variables X_{kj} , Y_{kj} and Z_{kj}.
XY_{kjj}	Continuous non-negative variable as a product of the decision variables X_{kj} and Y_{kj}.
L'_{ki,p_2}	Decision variable that takes the value of L_{ki} if that value is contained in the linearized segment p_2.
Auxiliary binary variables	
$v1_{kj,p_1}$	Decision variable that allows the objective function to include the value of the intercept of the linear equation of the segment p_1 for supplier k and DC j, if the value of NYV_{kj} is located at that segment p_1. Otherwise it will take the value of 0.
$v2_{ki,p_2}$	Decision variable that allows the objective function to include the value of the intercept of the linear equation of the segment p_2 for supplier k and DC i, if the value of L_{ki} is located at that segment p_2. Otherwise it will take the value of 0.

Table 3-4 Notations

$$\min \sum_{k\in K}\sum_{j\in J} F_j Z_{kj}\left(\psi \sum_{i\in I} \mu_{ki} Y_{kji}\right) + \sum_{k\in K}\sum_{j\in J} g_j X_{kj}(1-Z_{kj})(\psi \mu_{kj} Y_{kjj}) + \sum_{k\in K}\sum_{j\in J} w_{kj}\left(\psi \sum_{i\in I} \mu_{ki} Y_{kji}\right)$$
$$+ \sum_{j\in J}\sum_{i\in I} t_{ji}\left(\psi \sum_{k\in K} \mu_{ki} Y_{kji}\right) + \sum_{k\in K}\sum_{j\in J} H_j\left(\gamma \sum_{i\in I} \mu_{ki} Y_{kji}\right) + \sum_{j\in J}\sum_{i\in I} H_i\left(\gamma \sum_{k\in K} \mu_{ki} Y_{kji}\right)$$
$$+ z_\alpha \sum_{k\in K}\sum_{j\in J} H_j \sqrt{N_{kj}} \sqrt{\sum_{i\in I} \sigma_{ki}^2 Y_{kji}} + z_\alpha \sum_{i\in I}\sum_{k\in K} H_i \sqrt{L_{ki}} \sqrt{\sigma_{ki}^2} \tag{1}$$

Subject to:

$$N_{kj} \geq \left(l_{kj} + \gamma\right) - S_{kj}, \qquad \forall j, k \tag{2}$$

$$S_{kj} \leq X_{kj}\lambda - (\lambda - LT_{Max})X_{kj}Z_{kj}, \ \forall \ k \ , j \tag{3}$$

$$N_{kj} \leq LT_{Max}X_{kj}, \qquad \forall k, j \tag{4}$$

$$L_{ki} \geq \sum_{j \in J}\left(S_{kj} + (n_{kj} + \gamma)Y_{kji}\right) - \lambda, \ \forall k, j, i \tag{5}$$

$$L_{ki} \leq LT_{Max}(1 - X_{kj}) \ \forall k, i \tag{6}$$

$$\sum_{j \in J} Y_{kji} = 1, \ \forall i, k \tag{7}$$

$$Y_{kji} \leq X_{kj}, \ \forall i, k, j \tag{8}$$

$$Y_{kji} \leq Z_{kj}, \qquad \forall i, j, k \,\hat{}\, (i \neq j) \tag{9}$$

$$Z_{kj} \leq \sum_{(i,j) \wedge (i \neq j)} Y_{kji}\,, \ \forall k, j \tag{10}$$

$$Y_{kji} \in \{0,1\}, \ \forall i, j, k \tag{11}$$

$$X_{kj} \in \{0,1\}, \ \forall j, k \tag{12}$$

$$Z_{kj} \in \{0,1\}, \ \forall j, k \tag{13}$$

$$N_{kj} \geq 0 \ \ \forall j, k \tag{14}$$

$$L_{ki} \geq 0 \ \ \forall j, k \tag{15}$$

$$S_{kj} \geq 0, \ \forall k, j \tag{16}$$

In the objective function, the first two terms represent the facility cost where DC j is DC that receives merchandise from supplier k.The first term represents the cost where DC j is a consolidation DC while the second one indicates when the DC only handles its own demand. The third term refers to the transportation cost from suppliers to DC, and the fourth term covers the inter-DC transportation cost from CDCs to RDCs. The fifth term considers the pipeline inventory cost at CDCs, while the sixth one corresponds to the pipeline inventory cost at RDCs. The seventh term refers to the cost of holding safety stock at the DCs that receive merchandise from the suppliers. The amount of safety stock depends on the length of the Net Lead Time and the desired service level. The last term expresses the cost of the safety stock at RDCs that receive the merchandise from CDCs.

Constraint (2) ensures that the Net Lead Time for DC j that receive merchandise from supplier k is greater or equal to the difference between the time DC j needs to process a new receipt from supplier and the service time it requires to serve the DCs at the next tier. This is in line with Guaranteed Service Model as explained in Section 2.3. Constraint (3) makes sure that, in the case DC j is a consolidation center and receives merchandise directly from supplier k, its Net Lead Time can get a value bounded by a parameter LT_{MAX}. In case it is not a consolidation center, its maximum value is the service delivery time to the end customer. In case DC j does not receive any merchandise form supplier k, then by definition, it does not have any service time commitment to another DC and the value of S_{kj} must hence be zero. Constraint (4) makes sure that, if DC j does not get merchandise from supplier k, N_{kj} must be equal to zero as there would not be any lead time involving the supplier and that node.

Constraint (5) determines that the Net Lead Time for DC i that receives merchandise from DC j, as in constraint (2), must be greater or equal to the difference between the time DC i needs to process a new receipt from supplier and the service delivery time it requires to serve the end customer. Constraint (6) restricts this previous Net Lead Time to have non-zero value only if DC i doesn´t receive merchandise from supplier k. Constraint (7) makes sure that each DC i only gets shipments from a unique DC j. Constraint (8) allows DC j to serve demand of DC i, including itself (i.e., when *I* = *j*) , only if it receives merchandise from supplier k. Constraint (9) restricts shipping products from DC j to DC i only when DC j acts as CDC. Constraint (10) ensures that DC j can only be a CDC when it ships merchandise to other DCs. Constraints (11) to (15) are the standard definition of the variables to be either binary or continuous-valued.

3.3.3 Model Reformulation to Linearize

In this section, we discuss the steps taken to linearize the model developed in Section 3.2. The model in the objective function contains a couple of nonlinear terms such as products of decision variables, non-linear functions and square root terms. In order to linearize the model, we used piecewise linear approximation with sufficient number of segments to approximate the univariate concave terms.

The first step of linearization is replacing the product of two binary decision variables in objective function with new binary variables. Therefore, we replaced the following products with new variables:

$$Z_{kj} * Y_{kji} = YZ_{kji} \quad (17)$$

$$X_{kj} * Z_{kj} = XZ_{kj} \quad (18)$$

$$N_{kj} * Y_{kji} = NY_{kji} \quad (19)$$

where YZ_{kji}, XZ_{kj} and NY_{kji} are continuous and non-negative binary variables. To ensure that the added binary variables hold the same characteristics, the following new constraints are added:

$$YZ_{kji} \leq Y_{kji} \qquad \forall k, j, i \quad (20)$$

$$YZ_{kji} \leq Z_{kj} \qquad \forall k, j \quad (21)$$

$$YZ_{kji} \geq Y_{kji} + Z_{kj} - 1 \qquad \forall k, j, i \quad (22)$$

$$YZ_{kji} \geq 0 \quad \forall k, j, i \quad (23)$$

where constraints (20), (21) and (23) ensure that if Z_{kj} and Y_{kji} are zero, YZ_{kji} must be zero too. Constraint (22) ensures that if Z_{kj} and Y_{kji} are both equal to one, YZ_{kji} also takes the value of one.

In second objective function term there are products of the two binary decision variables as well - X_{kj} and Z_{kj}. They can be linearized by introducing an additional variable XZ_{kj} for both terms that are continuous non-negative. Similarly, in the case of X_{kj} and Y_{kji} we introduce variable XY_{kjj} as a product, and XYZ_{kjj} as a product of three variables: X_{kj}, Z_{kj} and Y_{kji}, with the following constraints:

$$XZ_{kj} \leq X_{kj}, \ \forall k, j \quad (24)$$

$$XZ_{kj} \leq Z_{kj}, \ \forall k, j \quad (25)$$

$$XZ_{kj} \geq X_{kj} + Z_{kj} - 1, \ \forall k, j \quad (26)$$

$$XY_{kjj} \leq X_{kj}, \ \forall k, j \quad (27)$$

$$XY_{kjj} \leq Y_{kjj}, \ \forall k, j \quad (28)$$

$$XY_{kjj} \geq X_{kj} + Y_{kjj} - 1, \ \forall k, j \quad (29)$$

$$XYZ_{kjj} \leq X_{kj}, \ \forall k, j \quad (30)$$

$XYZ_{kjj} \leq Y_{kjj},\ \forall k,j$ (31)

$XYZ_{kjj} \leq Z_{kj},\ \forall k,j$ (32)

$XYZ_{kjj} \geq X_{kj} + Y_{kjj} + Z_{kj} - 2,\ \forall\ k,j$ (33)

Constraints (24) and (25), ensure that if X_{kj} and Z_{kj} are zero, XZ_{kj} will be zero too. Constraint (26) ensures that if X_{kj} and Z_{kj} are both equal to one, XZ_{kj} also takes the value of one. In the same way constraint (27) and (28) ensure that if X_{kj} and Y_{kj} are zero, XY_{kjj} must be zero too. Constraint (29) ensures that if X_{kj} and Y_{kj} are both equal to one, XY_{kjj} also takes the value of one. In addition, constraints (30) to (32) ensure that if X_{kj}, Y_{kj} and Z_{kj} are zero, XYZ_{kjj} must be zero too. And finally, constraint (33) ensures that if X_{kj}, Y_{kj} and $Z_{kj}$are both equal to one, XYZ_{kj} also takes the value of one.

The same bilinear situation, with product of two decision variables N_{kj} and Y_{kji}, is present in the seventh term of the objective function. For that reason, NY_{kji} and $NY1_{kji}$ are introduced as new decision variables with their corresponding constraints:

$NY_{kji} + NY1_{kji} = N_{kj} \qquad \forall k,j,i$ (34)

$NY_{kji}\ \leq Y_{kji}\ LTmax \qquad \forall k,j,i$ (35)

$NY1_{kji} \leq (1 - Y_{kji}\ )LTmax \qquad \forall k,j,i$ (36)

$NY_{kji} \geq 0 \quad \forall k,j,i$ (37)

$NY1_{kji} \geq 0 \quad \forall k,j,i$ (38)

where constraints (36) forces $NY1_{kji}$ to zero when Y_{kji} equals one, and thereby constraint (34) ensures that NY_{kji} takes the value of N_{kj}. On the other hand, constraint (35) forces NY_{kji} to zero when Y_{kji} is equal to zero, thereby constraint (34) ensures that $NY1_{kji}$ takes the value of N_{kj}. In both cases, the constraints bind the value of NY_{kji} and $NY1_{kji}$ to LT_{max}, which is the upper bound for the variable N_{kj}.

In constraint (5) $L_{ki} \geq \sum_{j \in J}\left(S_{kj} + (n_{kj} + \gamma)Y_{kji}\right) - \lambda,\ \forall k,j,i$ there is a product of two decision variables S_{kj} and Y_{kji}. Because of that, we introduce SY_{kji} as a product of both variables along with the auxiliary variable $SY1_{kji}$ with the following constraints:

$$SY_{kji} + SY1_{kji} = S_{kj} \qquad \forall k, j, i \tag{39}$$

$$SY_{kji} \leq Y_{kji}\, LTmax \qquad \forall k, j, i \tag{40}$$

$$SY1_{kji} \leq (1 - Y_{kji}\,)LTmax \qquad \forall k, j, i \tag{41}$$

$$SY_{kji} \geq 0 \quad \forall k, j, i \tag{42}$$

$$SY1_{kji} \geq 0 \quad \forall k, j, i \tag{43}$$

As with the set of constraints that accompanied NY_{kji} and $NY1_{kji}$, constraints (39) to (43) ensure that $SY_{kji}$is assigned the value of $S_{kj}$when Y_{kji} equals one. On the other hand, $SY1_{kji}$is assigned the value of $S_{kj}$when Y_{kji} equals zero. Thanks to the above constraints, they bind the values of SY_{kji} and $SY1_{kji}$ to LT_{max}, which is the upper bound for the variable S_{kj}.

After replacing the new binary variables as mentioned, we have a new objective function (44).

$$\min \sum_{k\in K}\sum_{j\in J} F_j\psi \left(\sum_{i\in I}\mu_{ki}ZY_{kji}\right) + \sum_{k\in K}\sum_{j\in J} g_j\, X_{kj}\big(Y_{kjj} - YZ_{kjj}\big)(\psi\mu_{kj}) + \sum_{k\in K}\sum_{j\in J} w_{kj}\left(\psi\sum_{i\in I}\mu_{ki}Y_{kji}\right) + \sum_{j\in J}\sum_{i\in I} t_{ji}\left(\psi\sum_{k\in K}\mu_{ki}Y_{kji}\right) + \sum_{k\in K}\sum_{j\in J} H_j\left(\gamma\sum_{i\in I}\mu_{ki}Y_{kji}\right) + \sum_{j\in J}\sum_{i\in I} H_i\left(\gamma\sum_{k\in K}\mu_{ki}Y_{kji}\right) + z_\alpha\sum_{k\in K}\sum_{j\in J} H_j\sqrt{\sum_{i\in I}\sigma_{ki}^2 NY_{kji}} + z_\alpha\sum_{i\in I}\sum_{k\in K} H_i\sqrt{L_{ki}}\sqrt{\sigma_{ki}^2} \tag{44}$$

Now, to convert the square root terms into a linear combination that we can use in a MILP formulation, we use Piecewise Linear Approximation as suggested by You et al (2010). Before applying this technique, we define a new decision variable NYV_{kj} whose value is defined by the following constraint:

$$NYV_{kj} = \textstyle\sum_{i\in I}\sigma_{ki}^2 NY_{kji} \qquad \forall k, j \tag{45}$$

$$NYV_{kj} \geq 0\,, \quad \forall k, j \tag{46}$$

This way, the expression within the square root of the seventh term of the objective function can be reduced to just NYV_{kj}, which will ease the conversion to a piecewise linear function. One important fact that will be used later is that, as NYV_{kj} is a linear combination of variables that represent N_{kj} when $Y_{kji} = 1$, the biggest value that each of these can get is LT_{max}. Therefore, the domain that we must consider for NYV_{kj} must be from zero to $LT_{max} \sum_{i \in I} \sigma_{ki}^2$. Term L_{ki} within the squared root of the eight term is the net lead time at DC i, and its value ranges from zero to LT_{max}.

The process applied to linearize the model with piece-wise approximation is explained in detail in Appendix A along with respective constraints.

After applying piece-wise approximation, objective function can finally be restated with new variables as equation (47):

$$\begin{aligned}
\min \sum_{k \in K} \sum_{j \in J} F_j \psi \left(\sum_{i \in I} \mu_{ki} YZ_{kji} \right) + \sum_{k \in K} \sum_{i \in I} g_j \left(XY_{kjj} - XYZ_{kjj} \right) \psi \mu_{ki} + \sum_{k \in K} \sum_{j \in J} w_{kj} \left(\psi \sum_{i \in I} \mu_{ki} Y_{kji} \right) \\
+ \sum_{j \in J} \sum_{i \in I} t_{ji} \left(\psi \sum_{k \in K} \mu_{ki} Y_{kji} \right) + \sum_{k \in K} \sum_{j \in J} H_j \left(\gamma \sum_{i \in I} \mu_{ki} Y_{kji} \right) + \sum_{j \in J} \sum_{i \in I} H_i \left(\gamma \sum_{k \in K} \mu_{ki} Y_{kji} \right) \\
+ z_\alpha \sum_{k \in K} \sum_{j \in J} H_j \sum_{p_1 \in P_1} \left(F1_{kj,p_1} v1_{kj,p_1} + C1_{kj,p_1} NYV'_{kj,p_1} \right) \\
+ z_\alpha \sum_{k \in K} \sum_{i \in I} H_i \sqrt{\sigma_{ki}^2} \sum_{p_2 \in P_2} \left(F2_{ki,p_2} v2_{ki,p_2} + C2_{ki,p_2} L'_{ki,p_2} \right)
\end{aligned} \tag{47}$$

Refer to Appendix B for the final list of equations, which includes the objective functions and all the constraints.

4 RESULTS AND ANALYSIS

In this chapter, we discuss the tools we used to develop the model and analyze the results

4.1 Results

The linearized model in objective function (47) can now be solved using an optimization solver. In order to solve the model, we used Gurobi optimization solver in python environment. The model was run on a Lenovo ThinkPad X1 laptop with Intel Core i7 processor, 1.90GHz CPU and 8 GB RAM, coded and solved using Python 3.7 and Gurobi Optimizer version 9.0.1 build v9.0.1rc0 (win64).

The mathematical model was run for the three key product classes presented in Figure 3-5. We considered past one year's aggregated data. Data elements used are explained in Table 3-3. Service level *z*-score was set to current value as suggested by the company. Model was flexible to accept a single product class or multiple. As a product class consisted of multiple SKUs, each product class had more than one supplier as explained in Section 3.1. Gap parameter for Gurobi solver *MITGap* was set to 5 percent, which was an acceptable gap giving reasonable computation performance time. It generated around 328000 continuous variables, 328000 binary variables.

The model output computes the total logistics cost as well as the cost breakup for the recommended network configuration. Additionally, the details of the distribution network was presented separately for each product along with list of recommended CDCs. Distribution network is plotted on a map to provide visual aid for qualitative review and quicker decision.

The total supply chain costs of implementing the pooling strategy for the three product classes are tabulated in *Table 4-1*. The table also compares the supply chain cost of the proposed pooling strategy with the baseline scenario, which is Coppel's current operations cost without pooling. According to *Table 4-1*, the accumulated total cost of the three product lines is MXP $124 million. Comparing this cost with the accumulated cost of the baseline scenario, which is MXP $145 million, reveals that adopting pooling strategy could reduce total logistics cost by 14.63%, while maintaining the same service level (95%).

PRODUCT LINE	BASELINE	POOLING+GSM	% Savings	Service level	Computation time min
Washing Machine	$43,424	$38,020	12.44%	95%	10
Refrigerators	$65,168	$56,268	13.66%	95%	12
Television	$36,971	$29,978	18.91%	95%	9
Total	**$145,563**	**$124,266**	**14.63%**	**95%**	**90**

Table 4-1 Total annual cost comparison showing a 15% savings for top 3 products (MXP$1000)

Attaining these results was not surprising because adopting the pooling strategy would significantly reduce Coppel's two major supply chain costs: transportation cost from suppliers to DCs and the cost of keeping safety stock across the network. With restructuring of the supply chain network, supplier needs to ship merchandise to smaller number of DCs, and total safety stock is lower due to pooling effect. It is necessary to point that although adopting the new strategy may add the inter-DC transshipments cost as a new cost element to the supply chain network, total cost is lower compared the baseline scenario. On the other hand the pipeline inventory increased with pooling, and facility cost remain almost the same as seen in Table 4-2.

Product Line	Washing Machines		Refrigerators		Television	
CONCEPT COST	**Baseline**	**Pooling+GSM**	**Baseline**	**Pooling+GSM**	**Baseline**	**Pooling+GSM**
Consolidation facility	$0	$13,443	$0	$21,260	$0	$5,951
Regional DC facility	$15,334	$229	$24,628	$740	$6,857	$170
Supplier-DC transport	$26,766	$7,685	$37,835	$14,401	$27,195	$5,058
Inter DC transport	$0	$15,691	$0	$17,998	$0	$16,397
Supplier-DC pipeline inventory	$99	$87	$149	$138	$161	$190
Inter-DC pipeline inventory	$0	$77	$0	$104	$0	$146
Consolidation safety stock	$1,225	$641	$2,556	$1,406	$2,758	$1,053
Regional DC safety stock	$0	$167	$0	$221	$0	$1,013
TOTAL	**$43,424**	**$38,020**	**$65,168**	**$56,268**	**$36,971**	**$29,978**

Table 4-2 Comparison of annual cost breakup for two scenarios (MXP$1000).

In Table 4-3, we analyze the change by individual cost element. The main impact is on transportation cost with an 18.86% reduction and constitutes 63.06% share of the total annual logistics cost. Facility cost dropped by 12.03%, and constitutes 32.16% of the total logistics cost. Although safety stock cost got the biggest reduction of 45.28%, its share of total logistics cost is 4.49%. As mentioned earlier, pipeline inventory increased to meet GSM service level requirements, but it constitutes a small share of total logistics cost.

CONCEPT COST	Baseline	% Share	Pooling + GSM	% Change
Facility cost	$ 46,819	32.16%	$ 41,793	-12.03%
Transportation	$ 91,796	63.06%	$ 77,230	-18.86%
Pipeline inv	$ 409	0.28%	$ 742	44.88%
Safety Stock	$ 6,539	4.49%	$ 4,501	-45.28%
Total	**$ 145,563**	**100%**	**$ 124,266**	**-17.14%**

Table 4-3 Comparison of annual cost breakup change and share for two scenarios (MXP$1000)

We explain the model output with an example (Refrigerators product class and supplier 11347). Figure 4-1 shows the current state without any pooling. Supplier 11347, shown as a red dot, supplies to 19 Distribution centers marked as blue dots. Total annual cost MXP$10,737 includes $4,451 for facility cost, $5,793 for transportation, $32 for pipeline inventory and $461 for safety cost, as indicated in Table 4-4. Facility and transportation cost are the major contributors.

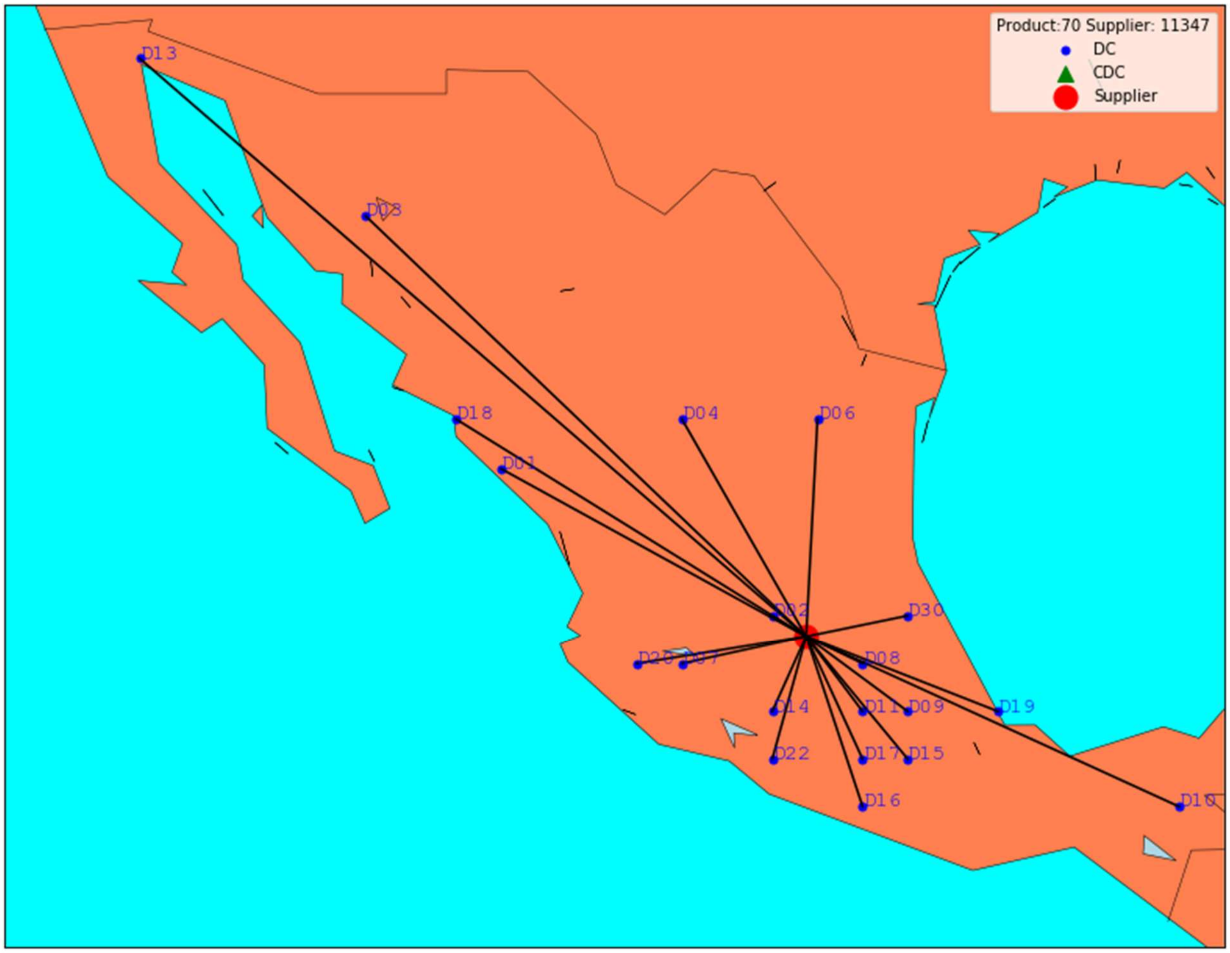

Figure 4-1 Model showing current network routes before consolidation

After running the mathematical model the network design proposed is represented in Figure 4-2. Supplier 11347 is shown in red dot and serves 10 distribution centers in first tier instead of 19 as was the case with baseline scenario. Six of those are pooling locations (CDCs) and are marked with green triangles. CDCs supply to 9 RDC, which are represented by the blue dot. In summary, first tier has 10 DC´s who receive directly from supplier; 4 of them fulfil their own demand while 6 DCs fulfil their demand as well as serve second tier DCs. In this example, we have 9 second tier DCs.

CONCEPT COST	Baseline	% Share	Pooling + GSM	% Change
Facility cost	$ 4,451	41.45%	$ 3,892	-14.36%
Transportation	$ 5,793	53.95%	$ 5,088	-13.86%
Pipeline inv	$ 32	0.30%	$ 52	38.46%
Safety Stock	$ 461	4.29%	$ 394	-17.01%
Total	**$ 10,737**	**100%**	**$ 9,426**	**-13.91%**

Table 4-4 Comparison of annual cost breakup for Refrigerators from supplier 11347 (MXP$1000)

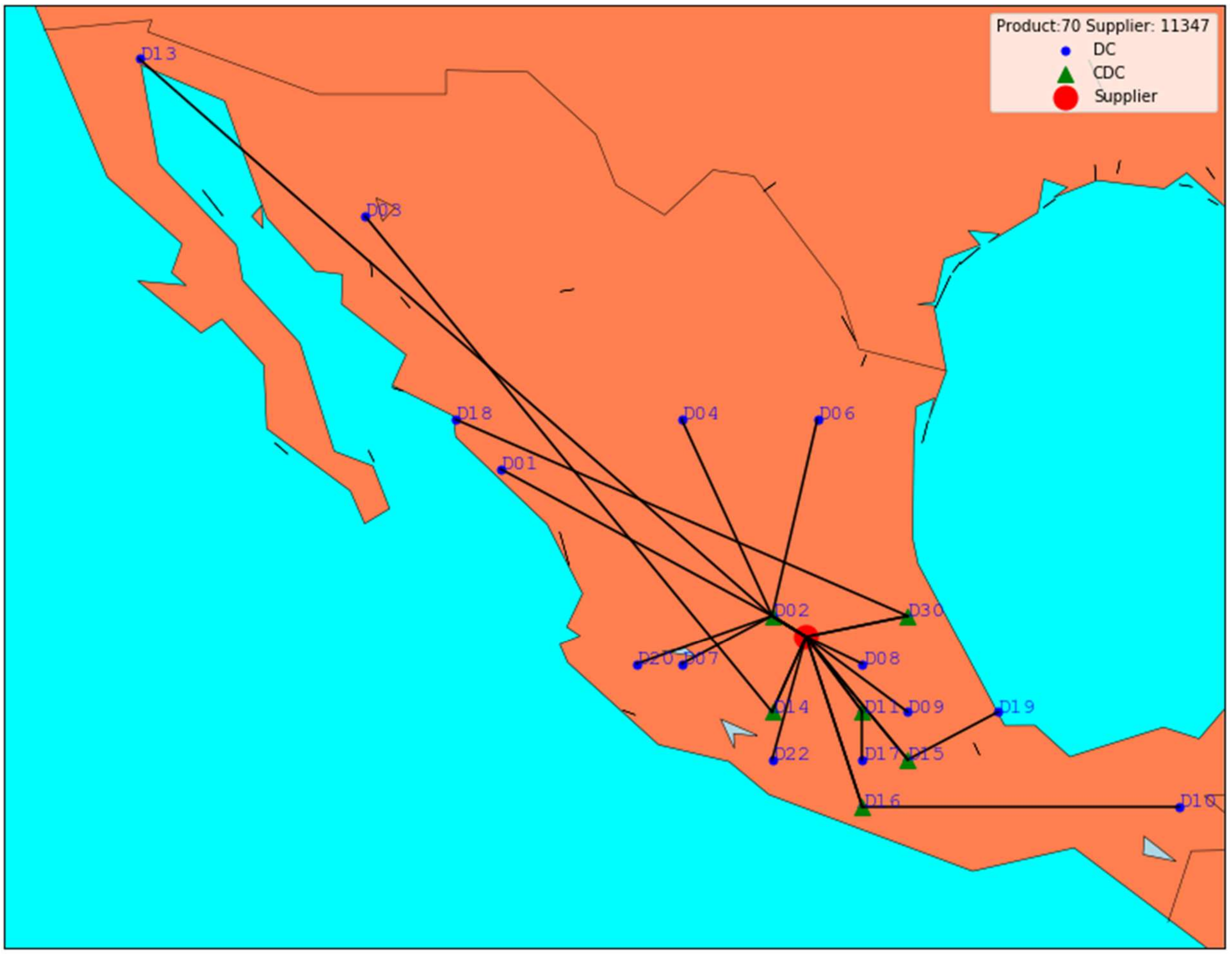

Figure 4-2 Model output showing routes after consolidation

The new configuration reduced the total transportation cost as seen in Table 4-4. This is primarily because the proposed network design requires supplier to ship to 10 locations instead of 19. This resulted in 13.86% saving in transportation cost, which contributes 53.95% to the total logistics cost. Facility cost is also lower 14.36% because storage and handling is cheaper in a CDC than in RDC. It contributes 41.45% to the total cost. Pipeline inventory increased by 38.46% but impact is not significant

(0.30%). Finally, safety stock cost reduced by 17% and has a lower share of total logistics cost. The recommended configuration would reduce the annual logistics cost by 13.91% for refrigerators from supplier 11347.

4.2 Model Limitations and Future Research

One of the main limitations of this model as per current formulation, is that it does not account for facility space or capacity constraints. However, the model can be easily extended to account for these restrictions. Also, the supplier lead time is assumed to be deterministic, which is often not true in reality.

We recommend further research and enhancement to the model to be able to handle more than two tiers within the multi-echelon distribution system. For instance, imported products are first received in Import DCs near the borders before being distributed to other DCs. Including Import DCs and retail stores in the model can significantly increase the size and complexity of the model, and potentially deteriorate model performance. Further research is recommended to evaluate the scalability of the solution to handle large problem size.

4.3 Insights and Management Recommendations

Pooling can help in large reduction in the inventory holding cost, by means of reducing the quantity of inventory at the lower tiers of the network. Such a decision affects the supplier and therefore requires good coordination with suppliers. Moreover, this change will result in shifting of inventory volumes between the DCs within the firm and requires careful change management and coordination between various stakeholders within the firm.

We have considered supplier's transportation cost in our optimization model even though supplier pays for the freight. Integrating partner's freight cost has allowed us to optimize at system level instead of locally within the firm. Firm can leverage this saving to renegotiate with supplier and arrive at a mutually beneficial arrangement.

Contingency plans must be evaluated in case an unforeseen event disrupts this network. As a principle of the Guaranteed Service Model approach, management needs to adopt extraordinary contingency measures for cases where demand is unexpectedly high or if there are other unforeseen supply disruptions in network. For instance, this may include measures to expedite shipments. As mentioned earlier, this model does not consider DC capacity as a constraint at present. Planners should review whether capacity is violated due to shift in volumes and any additional steps are required.

5 CONCLUSION

Successful firms continuously evaluate their supply chain operations and look for opportunities to uncover value. These firms are often willing to explore unconventional paths in search for such opportunities. These options must be objectively assessed to help make the right choices and necessary trade-offs. This project was one such attempt to explore opportunities to optimize cost by integrating both network and inventory decisions. We considered a novel perspective, where any of the existing DCs was made a candidate for pooling inventory at first-tier and distributing to other DCs that then become second-tier. One of the main challenges was to ensure that all the necessary costs and decision variables were factored in our model.

We developed an integrated mathematical model to determine the optimal network structure that could reduce the total logistics cost for a multi-echelon supply chain network with stochastic demand. The resultant model for this Location Inventory Problem was MINLP that we solved using Guaranteed Service Model approach. We used piecewise linear approximation technique to linearize the concave terms within the objective function.

Our results show that introducing pooling strategy can help with a 15% reduction in supply chain costs for some of the top product classes in company’s portfolio. Apart from reducing inventory cost, pooling can improve storage space utilization at the warehouse while maintaining the customer service levels.

Another key learning from this experience and our research is that we believe that integrated LIP techniques have not been properly harnessed. Improvement in computing power and modeling techniques has now made it simpler and computationally efficient to solve many such supply chain problems in a more integrated fashion than solving location and inventory problems separately in silos.

Even though this model was designed to solve a Location Inventory Problem in the context of a retail company, we believe that the model is robust enough to be used in other industries with little or no change.

REFERENCES

Baghalian, A., Rezapour, S., & Farahani, R. Z. (2013). Robust supply chain network design with service level against disruptions and demand uncertainties: A real-life case. *European Journal of Operational Research*, *227*(1), 199–215.

Bureau, US Census. (2020, February 20). Annual Retail Trade Survey: 2018. The United States Census Bureau. https://www.census.gov/data/tables/2018/econ/arts/annual-report.html

Diabat, A., Aouam, T., & Al-Araidah, O. (2009). The uncapacitated fixed-charge facility location problem with a multi-echelon inventory system. 2009 International Conference on Computers Industrial Engineering, 803–807.

eMarketer. (2019, May 17). Top 10 Retail Sites in Mexico, Ranked by Unique Visitors, March 2019 (millions). EMarketer. https://www.emarketer.com/chart/228549/top-10-retail-sites-mexico-ranked-by-unique-visitors-march-2019-millions

Eppen, G. D. (1979). Effects of Centralization on Expected Costs in a Multi-Location Newsboy Problem. *Management Science*, *25*(5), 498.

Eruguz, A. S., Sahin, E., Jemai, Z., & Dallery, Y. (2016). A comprehensive survey of guaranteed-service models for multi-echelon inventory optimization. *International Journal of Production Economics*, *172*, 110–125.

Euromonitor International. (2020, February). Retailing in Mexico-Analysis, Country Report. Retrieved from https://www.portal.euromonitor.com/

Farahani, R. Z., Rashidi Bajgan, H., Fahimnia, B., & Kaviani, M. (2015). Location-inventory problem in supply chains: A modelling review. *International Journal of Production Research*, *53*(12), 3769–3788.

Fisher, M. L. (1981). The Lagrangian Relaxation Method for Solving Integer Programming Problems on JSTOR. *Management Science*, *27*(1), 1–18.

Government of Canada, S. C. (2018, June 6). Manufacturers' sales, inventories, orders and inventory to sales ratios, by industry (dollars unless otherwise noted). https://www150.statcan.gc.ca/t1/tbl1/en/tv.action?pid=1610004701

Graves, S. C., & Willems, S. P. (2003). Supply Chain Design: Safety Stock Placement and Supply Chain Configuration. *Handbooks in Operations Research and Management Science*, *11*, 95–132.

Gupta, H. (2008). *Network design and safety stock placement for a multi-echelon supply chain* (Thesis, Massachusetts Institute of Technology). Retrieved from https://dspace.mit.edu/handle/1721.1/45227

Harahap, A., Mawengkang, H., & Opim, S. S. (2019). An Optimization Model for Multi-Echelon Multi-Commodity Supply Chain Design with Side Constraints. *Journal of Physics: Conference Series*

Hiassat, A., Diabat, A., & Rahwan, I. (2017). A genetic algorithm approach for location-inventory-routing problem with perishable products. *Journal of Manufacturing Systems*, *42*, 93–103.

Jeet, V., Kutanoglu, E., & Partani, A. (2009). Logistics network design with inventory stocking for low-demand parts: Modeling and optimization. IIE Transactions, 41(5), 389.

Kurata, H. (2014). How does inventory pooling work when product availability influences customers' purchasing decisions? *International Journal of Production Research*, *52*(22), 6739–6759.

Liao, S.-H., & Hsieh, C.-L. (2009). A Capacitated Inventory-Location Model: Formulation, Solution Approach and Preliminary Computational Results. In B.-C. Chien, T.-P. Hong, S.-M. Chen, & M. Ali (Eds.), Next-Generation Applied Intelligence (pp. 323–332). Springer.

Maister, D. H. (1976). Centralisation of Inventories and the "Square Root Law." International Journal of Physical Distribution & Logistics Management, 6(3), 124–134.

Miranda, P. A., & Garrido, R. A. (2009). Inventory service-level optimization within distribution network design problem. International Journal of Production Economics, 122(1), 276–285.

Nasiri, G. R., Davoudpour, H., & Karimi, B. (2010). The impact of integrated analysis on supply chain management: A coordinated approach for inventory control policy. Supply Chain Management, 15(4), 277–289.

Oeser, G., & Romano, P. (2016). An empirical examination of the assumptions of the Square Root Law for inventory centralisation and decentralisation. International Journal of Production Research, 54(8), 2298–2319.

Pagh, J. D., & Cooper, M. C. (1998). Supply Chain Postponement and Speculation Strategies: How to Choose the Right Strategy. *Journal of Business Logistics*, *19*(2), 13–33.

Schmidt, G., & Wilhelm, W. E. (2000). Strategic, tactical and operational decisions in multi-national logistics networks: A review and discussion of modelling issues. *International Journal of Production Research*, 38(7), 1501–1523.

Schuster Puga, M., & Tancrez, J.-S. (2017). A heuristic algorithm for solving large location–inventory problems with demand uncertainty. European Journal of Operational Research, 259(2), 413–423.

Shen, Z.-J. M., Coullard, C., & Daskin, M. S. (2003). A Joint Location-Inventory Model. Transportation Science, 37(1), 40.

Yao, Z., Lee, L. H., Jaruphongsa, W., Tan, V., & Hui, C. F. (2010). Multi-source facility location–allocation and inventory problem. European Journal of Operational Research, 207(2), 750–762.

You, F., & Grossmann, I. E. (2010). Integrated Multi-Echelon Supply Chain Design with Inventories Under Uncertainty: MINLP Models, Computational Strategies. *AIChE Journal*, *56*(2), 419–440.

Zheng, X., Yin, M., & Zhang, Y. (2019). Integrated optimization of location, inventory and routing in supply chain network design. *Transportation Research Part B: Methodological*, *121*, 1–20.

Appendix A: Parameter Definition for Piece Wise Linear Approximation.

In order to define interception and slope parameters for linear segments of the approximation (You & Grossman, 2010), this section presents the procedures for finding the values of these.

For the seventh term of the Objective Function (47) $\sqrt{NYV_{kj}}$, we split the domain into n_1 equal parts. The cutting points for each lineal segment are contained in the set $M1$ of $n_1 + 1$ elements, as shows equation 48.

$$M1 = \left\{0, \frac{LT_{max}}{n_1} \sum_{i \in I} \sigma_{ki}^2, \frac{2LT_{max}}{n_1} \sum_{i \in I} \sigma_{ki}^2, \dots, \frac{(n_1 - 1)LT_{max}}{n_1} \sum_{i \in I} \sigma_{ki}^2, LT_{max} \sum_{i \in I} \sigma_{ki}^2 \right\} \tag{48}$$

Defining $p_1 \in P_1 = \{1,2, \dots, n_1 - 1, n_1\}$ as the indicator of which segment we refer to in the piecewise linear function after dividing the original function into n_1 linear segments, we can define the slope $C1_{kj,p_1}$ of each segment as follows in Equation 49.

$$\begin{aligned} C1_{kj,p_1} &= \frac{\sqrt{M1_{p_1}} - \sqrt{M1_{p_1-1}}}{M1_{p_1} - M1_{p_1-1}} = \frac{\sqrt{\frac{p_1 LT_{max}}{n_1} \sum_{i \in I} \sigma_{ki}^2} - \sqrt{\frac{(p_1 - 1)LT_{max}}{n_1} \sum_{i \in I} \sigma_{ki}^2}}{\frac{p_1 LT_{max}}{n_1} \sum_{i \in I} \sigma_{ki}^2 - \frac{(p_1 - 1)LT_{max}}{n_1} \sum_{i \in I} \sigma_{ki}^2} \\ &= \frac{\sqrt{\frac{LT_{max}}{n_1} \sum_{i \in I} \sigma_{ki}^2} \left(\sqrt{p_1} - \sqrt{p_1 - 1}\right)}{\frac{LT_{max}}{n_1} \sum_{i \in I} \sigma_{ki}^2} = \sqrt{\frac{n_1}{LT_{max} \sum_{i \in I} \sigma_{ki}^2}} \left(\sqrt{p_1} - \sqrt{p_1 - 1}\right) \end{aligned} \tag{49}$$

and the intercept $F1_{kj,p_1}$ of each segment is presented in Equation 50.

$$
\begin{aligned}
F1_{kj,p_1} &= \sqrt{M1_{p_1-1}} - C1_{kj,p_1} M1_{p_1-1} \\
&= \sqrt{\frac{(p_1-1)LT_{max}}{n_1}\sum_{i\in I}\sigma_{ki}^2} \\
&\quad - \sqrt{\frac{n_1}{LT_{max}\sum_{i\in I}\sigma_{ki}^2}}\left(\sqrt{p_1}-\sqrt{p_1-1}\right)\left(\frac{(p_1-1)LT_{max}}{n_1}\sum_{i\in I}\sigma_{ki}^2\right) \\
&= \sqrt{\frac{LT_{max}}{n_1}\sum_{i\in I}\sigma_{ki}^2}\sqrt{p_1-1} - \sqrt{\frac{LT_{max}}{n_1}\sum_{i\in I}\sigma_{ki}^2}\,(p_1-1)\left(\sqrt{p_1}-\sqrt{p_1-1}\right) \\
&= \sqrt{\frac{LT_{max}}{n_1}\sum_{i\in I}\sigma_{ki}^2}\left(\sqrt{p_1-1}-(p_1-1)\left(\sqrt{p_1}-\sqrt{p_1-1}\right)\right)
\end{aligned}
$$

(50)

Therefore, the squared root term $\sqrt{NYV_{kj}}$ would be approximated by the following expression:

$$\sum_{p_1\in P_1}\left(F1_{kj,p_1} v1_{kj,p_1} + C1_{kj,p_1} NYV'_{kj,p_1}\right)$$

(51)

Where NYV'_{kj,p_1} is a continuous decision variable that gets the value of NYV_{kj} only if that value is contained within the segment p_1. The binary decision variable $v1_{kj,p_1}$ represents whether the intercept of the equation of the linear segment p_1 must be used or not within the objective function. Both previous variables are regulated by the following constraints:

$$\textstyle\sum_{p_1\in P_1} v1_{kj,p_1} = 1\ ,\ \forall k,j \tag{52}$$

$$\textstyle\sum_{p_1\in P_1} NYV'_{kj,p_1} = NYV_{kj}\ ,\ \forall k,j \tag{53}$$

$$\tfrac{LT_{MAX}(p_1-1)}{n_1}\textstyle\sum_{i\in I}\sigma_{ki}^2\, v1_{kj,p_1} \le NYV'_{kj,p_1}\ ,\ \forall k,j,p_1 \tag{54}$$

$$\tfrac{LT_{MAX}p_1}{n_1}\textstyle\sum_{i\in I}\sigma_{ki}^2\, v1_{kj,p_1} \ge NYV'_{kj,p_1}\ ,\ \forall k,j,p_1 \tag{55}$$

$$v1_{kj,p_1} \in \{0,1\},\ \forall k,j,p_1 \tag{56}$$

$$NYV'_{kj,p_1} \ge 0,\ \forall k,j,p_1 \tag{57}$$

Constraint (53) forces the sum of all NYV'_{kj,p_1} to be equal to the value of NYV_{kj}. Therefore, by the nature of the approximation, for all $p_1 \in P_1$ segments, only one can have a corresponding nonzero NYV'_{kj,p_1}. This is also forced by constraint (52), where only one intercept can have an effect in the objective function and constraints (54) and (55) bound the value of NYV'_{kj,p_1} to the respective cutting points of that segment multiplied by $v1_{kj,p_1}$: if this last variable is zero for the m-th segment, then $NYV'_{kj,m}$ must be equal to zero and must not contribute to the objective function.

The same procedure is done for the eight term of the objective function. For the expression$\sqrt{l_{KI}}$, we will split the domain into n_2 equal parts. The cutting points for each lineal segment are contained in the set $M2$ of $n_2 + 1$ elements in Equation (58).

$$M2 = \left\{0, \frac{LT_{max}}{n_2}, \frac{2LT_{max}}{n_2}, \dots, \frac{(n_2 - 1)LT_{max}}{n_2}, LT_{max}\right\} \tag{58}$$

Defining $p_2 \in P_2 = \{1,2, \dots, n_2 - 1, n_2\}$ as the indicator of which segment we refer to in the piecewise linear function after dividing the original function into n_2 linear segments; we can define the slope $C2_{ki,p_2}$ of each segment as follows in Equation (59).

$$C2_{ki,p_1} = \frac{\sqrt{M2_{p_2}} - \sqrt{M2_{p_2-1}}}{M2_{p_2} - M2_{p_2-1}} = \frac{\sqrt{\frac{p_2 LT_{max}}{n_2}} - \sqrt{\frac{(p_2-1)LT_{max}}{n_2}}}{\frac{p_2 LT_{max}}{n_2} - \frac{(p_2-1)LT_{max}}{n_2}} = \frac{\sqrt{\frac{LT_{max}}{n_2}}\left(\sqrt{p_2} - \sqrt{p_2 - 1}\right)}{\frac{LT_{max}}{n_2}}$$
$$= \sqrt{\frac{n_2}{LT_{max}}}\left(\sqrt{p_2} - \sqrt{p_2 - 1}\right) \tag{59}$$

and the intercept $F2_{ki,p_2}$ of each segment as Equation (60) explains.

$$F2_{ki,p_2} = \sqrt{M2_{p_2-1}} - C2_{ki,p_2}M2_{p_2-1}$$
$$= \sqrt{\frac{(p_2-1)LT_{max}}{n_2}} - \sqrt{\frac{n_2}{LT_{max}}}\left(\sqrt{p_2} - \sqrt{p_2-1}\right)\left(\frac{(p_2-1)LT_{ma}}{n_2}\right)$$
$$= \sqrt{\frac{LT_{max}}{n_2}}\sqrt{p_2-1} - \sqrt{\frac{LT_{max}}{n_2}}(p_2-1)\left(\sqrt{p_2} - \sqrt{p_2-1}\right)$$
$$= \sqrt{\frac{LT_{max}}{n_2}}\left(\sqrt{p_2-1} - (p_2-1)\left(\sqrt{p_2} - \sqrt{p_2-1}\right)\right)$$

(60)

Hence, $\sqrt{L_{ki}}$ would be approximated by the following expression in Equation (61).

$$\sum_{p_2 \in P_2} \left(F2_{ki,p_2} v2_{ki,p_2} + C2_{ki,p_2} L'_{ki,p_2}\right)$$

(61)

Where, as in the previous approximation, L'_{ki,p_2} is a continuous decision variable that gets the value of L_{ki} only if that value is contained within the segment p_2. The binary decision variable $v2_{ki,p_2}$ represents whether the intercept of the equation of the linear segment p_2 must be used or not within the objective function. Their related constraints are shown below:

$\sum_{p_2 \in P_2} v2_{ki,p_2} = 1 \;, \; \forall k, i$ (62)

$\sum_{p_2 \in P_2} L'_{ki,p_2} = L_{ki} \;, \; \forall k, i$ (63)

$\frac{LT_{MAX}(p_2-1)}{n_2} v2_{ki,p_2} \le L'_{ki,p_2} \;, \; \forall k, i, p_2$ (64)

$\frac{LT_{MAX}p_2}{n_2} v2_{ki,p_2} \ge L'_{ki,p_2}, \; \forall k, i, p_2$ (65)

$v2_{ki,p_2} \in \{0,1\} \;, \; \forall k, i, p_2$ (66)

$L'_{ki,p_2} \ge 0 \;, \; \forall k, i, p_2$ (67)

Constraints (62) to (67) work the same as their counterparts for the approximation of $\sqrt{NYV_{kj}}$.

Final Objective Function (47) with new variables and piece-wise approximation method.

$$\min \sum_{k\in K}\sum_{j\in J} F_j\psi\left(\sum_{i\in I}\mu_{ki}YZ_{kji}\right) + \sum_{k\in K}\sum_{i\in I} g_j\left(XY_{kjj} - XYZ_{kjj}\right)\psi\mu_{ki} + \sum_{k\in K}\sum_{j\in J} w_{kj}\left(\psi\sum_{i\in I}\mu_{ki}Y_{kji}\right)$$
$$+ \sum_{j\in J}\sum_{i\in I} t_{ji}\left(\psi\sum_{k\in K}\mu_{ki}Y_{kji}\right) + \sum_{k\in K}\sum_{j\in J} H_j\left(\gamma\sum_{i\in I}\mu_{ki}Y_{kji}\right) + \sum_{j\in J}\sum_{i\in I} H_i\left(\gamma\sum_{k\in K}\mu_{ki}Y_{kji}\right)$$
$$+ z_\alpha \sum_{k\in K}\sum_{j\in J} H_j \sum_{p_1\in P_1}\left(F1_{kj,p_1}v1_{kj,p_1} + C1_{kj,p_1}NYV'_{kj,p_1}\right)$$
$$+ z_\alpha \sum_{k\in K}\sum_{i\in I} H_i\sqrt{\sigma_{ki}^2}\sum_{p_2\in P_2}\left(F2_{ki,p_2}v2_{ki,p_2} + C2_{ki,p_2}L'_{ki,p_2}\right)$$

(47)

Where:

$$F1_{kj,p_1} = \sqrt{\frac{LT_{MAX}}{n_1}\sum_{i\in I}\sigma_{ki}^2}\left(\sqrt{p_1-1} - (p_1-1)\left(\sqrt{p_1} - \sqrt{p_1-1}\right)\right),$$

$$C1_{kj,p_1} = \sqrt{\frac{n_1}{LT_{MAX}\sum_{i\in I}\sigma_{ki}^2}}\left(\sqrt{p_1} - \sqrt{p_1-1}\right)$$

$$F2_{ki,p_2} = \sqrt{\frac{LT_{MAX}}{n_2}}\left(\sqrt{p_2-1} - (p_2-1)\left(\sqrt{p_2} - \sqrt{p_2-1}\right)\right), \qquad C2_{ki,p_2} = \sqrt{\frac{n_2}{LT_{MA}}}\left(\sqrt{p_2} - \sqrt{p_2-1}\right)$$

Appendix B: Final Objective Formulation with Piecewise Linear Approximation

Presenting the final objective function (47) with all constraints in a single document to have the summary, the steps to get to this point where presented in detail in section from 3.3.2 to 3.3.3 and Appendix A.

$$
\begin{aligned}
\min \sum_{k\in K}\sum_{j\in J} F_j \psi\left(\sum_{i\in I}\mu_{ki} YZ_{kji}\right) + \sum_{k\in K}\sum_{j\in J} g_j\left(XY_{kjj} - XYZ_{kjj}\right)\psi\mu_{kj} + \sum_{k\in J}\sum_{j\in J} w_{kj}\left(\psi\sum_{i\in I}\mu_{ki} Y_{kji}\right) \\
+ \sum_{j\in J}\sum_{i\in I} t_{ji}\left(\psi\sum_{k\in K}\mu_{ki}Y_{kji}\right) + \sum_{k\in K}\sum_{j\in J} H_j\left(\gamma\sum_{i\in I}\mu_{ki}Y_{kji}\right) + \sum_{(i,j)\wedge(i\neq j)} H_i\left(\gamma\sum_{k\in K}\mu_{ki}Y_{kji}\right) \\
+ z_\alpha \sum_{k\in K}\sum_{j\in J} H_j \sum_{p_1\in P_1}\left(F1_{kj,p_1} v1_{kj,p_1} + C_{kj,p_1} NYV'_{kj,p_1}\right) \\
+ z_\alpha \sum_{k\in K}\sum_{i\in I} H_i\sqrt{\sigma_{ki}^2}\sum_{p_2\in P_2}\left(F2_{ki,p_2} v2_{ki,p_2} + C2_{ki,p_2} L'_{ki,p_2}\right)
\end{aligned}
\qquad (47)
$$

Where:

$$F1_{kj,p_1} = \sqrt{\frac{LT_{MAX}}{n_1}\sum_{i\in I}\sigma_{ki}^2}\left(\sqrt{p_1 - 1} - (p_1 - 1)\left(\sqrt{p_1} - \sqrt{p_1 - 1}\right)\right),$$

$$C1_{kj,p_1} = \sqrt{\frac{n_1}{LT_{MAX}\sum_{i\in I}\sigma_{ki}^2}}\left(\sqrt{p_1} - \sqrt{p_1 - 1}\right)$$

$$F2_{ki,p_2} = \sqrt{\frac{LT_{MAX}}{n_2}}\left(\sqrt{p_2 - 1} - (p_2 - 1)\left(\sqrt{p_2} - \sqrt{p_2 - 1}\right)\right), \qquad C2_{ki,p_2} = \sqrt{\frac{n_2}{LT_{MAX}}}\left(\sqrt{p_2} - \sqrt{p_2 - 1}\right)$$

Subject to:

Constraints for piecewise linear approximation

(52) $\sum_{p_1\in P_1} v1_{kj,p_1} = 1, \qquad \forall k, j$

(53) $\sum_{p_1\in P_1} NYV'_{kj,p_1} = NYV_{kj}, \qquad \forall k, j$

(54) $\left[\frac{LT_{MAX}(p_1 - 1)}{n_1}\sum_{i\in I}\sigma_{ki}^2\right] v1_{kj,p_1} \leq NYV'_{kj,p_1}, \qquad \forall k, j, p_1$

(55) $\left[\frac{LT_{MAX}p_1}{n_1}\sum_{i\in I}\sigma_{ki}^2\right] v1_{kj,p_1} \geq NYV'_{kj,p_1}, \qquad \forall k, j, p_1$

(62) $\sum_{p_2 \in P_2} v2_{ki,p_2} = 1, \qquad \forall k, i$

(63) $\sum_{p_2 \in P_2} L'_{ki,p_2} = L_{ki}, \qquad \forall k, i$

(64) $\left[\frac{LT_{MAX}(p_2 - 1)}{n_2}\right] v2_{ki,p_2} \leq L'_{ki,p_2}, \qquad \forall k, i, p_2$

(65) $\left[\frac{LT_{MAX} p_2}{n_2}\right] v2_{ki,p_2} \geq L'_{ki,p_2}, \qquad \forall k, i, p_2$

(45) $NYV_{kj} = \sum_{i \in I} \sigma_{ki}^2 NY_{kji}\,, \qquad \forall k, j$

Constraints for net lead time in both tiers

(2) $N_{kj} \geq (l_{kj} + \gamma) X_{kj} - S_{kj}, \qquad \forall k, j$

(3) $S_{kj} \leq X_{kj}\lambda - (\lambda - LT_{MAX}) XZ_{kj}, \qquad \forall k, j$

(4) $N_{kj} \leq LT_{MAX} X_{kj}, \qquad \forall k, j$

(5) $L_{ki} \geq \sum_{j \in J} \left(SY_{kji} + (n_{ji} + \gamma) Y_{kji} \right) - \lambda, \qquad \forall k, i$

(6) $L_{kj} \leq LT_{MAX}(1 - X_{kj}), \qquad \forall k, j$

Constraints for linearizing bilinear terms

(39) $SY_{kji} + SY1_{kji} = S_{kj}, \qquad \forall k, j, i$

(40) $SY_{kji} \leq LT_{MAX} Y_{kji}, \qquad \forall k, j, i$

(41) $SY1_{kji} \leq LT_{MAX}(1 - Y_{kji}), \qquad \forall k, j, i$

(20) $YZ_{kji} \leq Y_{kji}, \qquad \forall k, j, i$

(21) $YZ_{kji} \leq Z_{kj}, \qquad \forall k, j, i$

(22) $YZ_{kji} \geq Y_{kji} + Z_{kj} - 1, \qquad \forall k, j, i$

(24) $XZ_{kj} \leq X_{kj}, \qquad \forall k, j$

(25) $XZ_{kj} \leq Z_{kj}, \qquad \forall k, j$

(26) $XZ_{kj} \geq X_{kj} + Z_{kj} - 1, \qquad \forall k, j$

(27) $XY_{kjj} \leq X_{kj}, \qquad \forall k, j$

(28) $XY_{kjj} \leq Y_{kjj}, \qquad \forall k, j$

(29) $XY_{kjj} \geq X_{kj} + Y_{kjj} - 1, \qquad \forall k, j$

(30) $XYZ_{kjj} \leq X_{kj}, \qquad \forall k, j$

(31) $XYZ_{kjj} \leq Y_{kjj}, \qquad \forall k, j$

(32) $XYZ_{kjj} \leq Z_{kj}, \qquad \forall k, j$

(33) $XYZ_{kjj} \geq X_{kj} + Y_{kjj} + Z_{kj} - 2, \qquad \forall k, j$

Network design constraints

(34) $NY_{kji} + NY1_{kji} = N_{kj}, \qquad \forall k, j, i$

(35) $NY1_{kji} \leq LT_{MAX} Y_{kji}, \qquad \forall k, j, i$

(36) $NY1_{kji} \leq LT_{MAX}(1 - Y_{kji}), \qquad \forall k, j, i$

(7) $\sum_{j \in J} Y_{kji} = 1, \qquad \forall k, i$

(8) $Y_{kji} \leq X_{kj}, \qquad (\forall k, j, i) \wedge (i \neq j)$

(23.5) $Y_{kjj} = X_{kj}, \qquad \forall k, j$

(9) $Y_{kji} \leq Z_{kj}, \qquad (\forall k, j, i) \wedge (i \neq j)$

(10) $Z_{kj} \leq \sum_{(i,j) \wedge (i \neq j)} Y_{kji}\,, \qquad \forall k, j$

<u>Variables definitions: Piecewise linear approximation</u>

(56) $v1_{kj,p_1} \in \{0,1\}, \qquad \forall k, j, p_1$

(57) $NYV'_{kj,p_1} \geq 0, \qquad \forall k, j, p_1$

(66) $v2_{ki,p_2} \in \{0,1\}, \qquad \forall k, i, p_2$

(67) $L'_{ki,p_2} \geq 0, \qquad \forall k, i, p_2$

(46) $NYV_{kj} \geq 0, \qquad \forall k, j$

<u>Variables definition: Network design</u>

(11) $Y_{kji} \in \{0.1\}, \qquad \forall k, j, i$

(12) $X_{kj} \in \{0,1\}, \qquad \forall k, j$

(13) $Z_{kj} \in \{0,1\}, \qquad \forall k, j$

(14) $N_{kj} \geq 0, \qquad \forall k, j$

(15) $L_{ki} \geq 0, \qquad \forall k, i$

(16) $S_{kj} \geq 0, \qquad \forall k, j$

<u>Variables definition: Linearization of bilinear products</u>

(17) $YZ_{kji} \in \{0,1\}, \qquad \forall k, j, i$

$(17a)$ $XY_{kjj} \in \{0,1\}, \qquad \forall k, j$

(18) $XZ_{kj} \in \{0,1\}, \qquad \forall k, j$

$(18a)$ $XYZ_{kjj} \in \{0,1\}, \qquad \forall k, j$

(42) $SY_{kji} \geq 0, \qquad \forall k, j, i$

(43) $SY1_{kji} \geq 0, \qquad \forall k, j, i$

(37) $NY_{kji} \geq 0, \qquad \forall k, j, i$

(38) $NY1_{kji} \geq 0, \qquad \forall k, j, i$