Optimal Inventory Model for Managing Demand-Supply Mismatches for Perishables with Stochastic Supply

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Topic Areas: Inventory, Optimization, Sourcing

Summary: While festivals bring a reason to cheer for everyone, businesses dealing with a spike in demand for perishables may have to live with the misery of lost sales and/or expired items. As a case study, we look into the dairy industry which deals with liquid milk, where both raw material and finished goods are perishable, which implies that merely stockpiling inventory of either item, without paying attention to potential inventory losses, cannot be an optimal strategy. In this research, we formulate a stochastic optimization model to determine the supply targets for inventory planning that leads to optimal profits over the time-series.

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KEY INSIGHTS

1. **Presence of a large number of small suppliers leads to high supply variability. Festivals lead to a reduction in supply as suppliers increase personal consumption.**
2. **We can optimize the expected profit over a time series by determining supply targets and production targets at time period level.**
3. **It is not easy to increase/decrease suppliers at very short notice to manage shortages. This means we need to set supply targets at time series level, which reduces the overall profitability.**

Introduction

Effective inventory control policies drive supply chain efficiency and reduce costs, and thus continue to be a leading area of operations research at both academic and practitioner levels. Businesses use safety stock of finished goods as one of the tools to manage demand variability. Similarly, one of the options for dealing with supply uncertainty is also to carry safety stock in the form of finished goods and/or raw materials. While the lifetime of safety stock is usually assumed to be infinite, this is not the case with perishable items, which results in loss of inventory. In supply chains with capacity limitations, demand surges lead to intermittent demand-supply mismatches, which have a detrimental effect on any company in the area of operations and inventory. In this research, we address the unique problem of inventory management for perishables with supply variability, specifically the scenario of dealing with intermittent demand surges.

Seasonal events like festivals or special occasions often result in demand surges for some commodities. While this scenario should be a cause for cheer among most companies, the inability to service the increased demand might lead to lost sales, besides impacting customer goodwill. The food supply chain in developing countries like India involves a large number of small suppliers. At the individual level, these suppliers deal with capacity variability as well as yield variability. This research explores the dairy industry in Eastern India, where individual farmers form the bulk of the supplier base. These individuals offer surplus milk for sale to companies on a daily basis. Milk-based sweets form an integral part of festivities in India. In this scenario, the surge in demand across the communities during festivals leads to an intermittent gap between customer intake and supplier offtake around the same time. While safety stock would usually serve the need to manage variability in supply and demand, in the case of perishables, stocking raw milk and/or packed milk may result in inventory losses.

An extensive literature review indicates that while supply chain for perishables is well researched, they mostly deal with demand variability, and thus the primary focus is on inventory policies for finished goods. Supply variability for perishables is primarily researched from the perspective of stochastic delivery timelines and supply disruption. Mismatches in demand-supply are mainly handled via information dissemination as well as risk transference. Unfortunately, the unique combination of all three scenarios (perishability, supply variability, and demand surge) faced by entities dealing with perishables is not well researched and forms the core focus of this thesis. While this research centers around milk as a perishable commodity, the same can be generalized to other perishables with supply variability, such as fruits, vegetables, flowers, etc., where the primary supplier base consists of individuals rather than corporate/professional entities.

Research Methodology

Both qualitative and quantitative methods have been used to conduct the research. The process flow of the research methodology is depicted as follows:

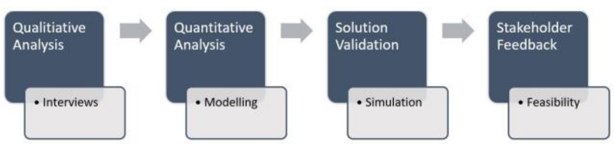

Figure 1: Research Methodology

As an outcome of interviews with stakeholders in the procurement process, we model the flow for raw material as follows.

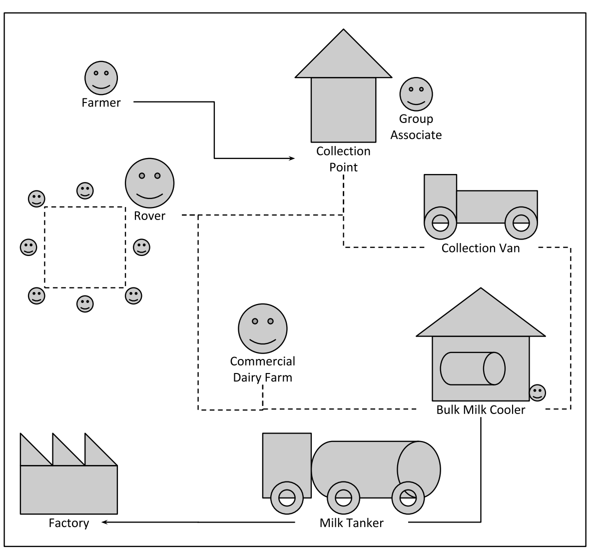

Figure 2: Process Flow for Milk Procurement

Farmers within the company’s network pour in milk at collection points, which are managed by Group Associates. Rovers collect milk from different farmers outside the company network. The milk collection van gathers milk from collection points, rovers, and commercial dairy farms to be brought over to Bulk Milk Cooler (BMC) for cooling and storage. This process is done twice a day, as milk extraction happens in morning and evening. In some cases, rovers or farmers may directly bring milk to the BMC. Once a day, the milk from BMC is brought over to the factory for storage and processing.

Stochastic Optimization Model

Our objective is to maximize profits while considering the potential losses on account of perishability and lost sales. In this step, we formulate the optimization problem, while realizing the stochastic nature of both demand and supply. We incorporate the impact of demand surge and supply reduction on specific time periods within the time series. This is indicative of the demand-supply mismatches expected around the specific time periods, i.e., festivals. We assume that the product is shipped to distributors based on firm orders received. To reflect the real-world situation, we model the need to specify the supply targets before we plan for production. Also, production needs to be based on the actual supply received, which is variable. Finally, we model the need to produce finished goods before knowing the actual order.

We make a key assumption with respect to perishability here as follows. The raw material received at the beginning of the time period needs to be consumed within the same time period. Any excess raw material not used in production needs to be discarded. Also, the finished goods produced during the time period needs to be shipped at the end of the time period. Any excess finished goods not needed to fulfill demand needs to be discarded. This situation closely represents the perishability challenges in the dairy supply chain

Table 1: Time Series for Supply-Production-Demand

Time Period	Supply	Raw Material	Production	Finished Goods	Demand
…	…				
99	500	…	…	…	
100	495	500	500	500	…
101	505	495	495	495	505
102	…	505	505	505	500
103		…	…	…	495
…					…

We note that during time period 101, production receives supply (495) that materialized during period 100, and produces finished goods (495) to cater to demand (500) during time period 102. As we can see, the time period of 101 results in lost sales, while time period 102 results in excess items.

We develop an optimization model with overall profit as the objective function and use a three-stage stochastic optimization approach. We address the scenario where supply targets can be set at time-period level, as well as at entire time-series level.

Table 2: Model Notation

Symbol	Explanation
SETS	
T	Set of time periods within the time series, indexed by t
I	Set of occurrences of a discrete probability of supply, indexed by i
J	Set of occurrences of a discrete probability of demand, indexed by j
PARAMETERS	
ΔD	Time periods between production and demand realization
ΔS	Time periods between supply realization and production
D	Demand expected per time period for the entire time series
C_1	The incremental cost of procurement of per unit of raw material
C_2	The incremental cost of production of per unit of finished goods
C_3	The incremental cost of selling per unit of finished goods to fulfill the demand
G_1	Salvage value per unit of excess raw material
G_2	Salvage value per unit of excess finished goods
P	Selling price realized for any unit sold
B	Shortage cost of unfulfilled demand per unit
p_i^S	Probability of the occurrence of a specific value of supply
δ_i^S	Scaling factor for the specific value of supply
p_j^D	Probability of the occurrence of a specific value of demand
δ_j^D	Scaling factor for the specific value of demand
Y_t	Binary value representing whether a specific time period has a simultaneous demand surge and supply shortage
α^D	The incremental surge in demand (+ve) during the relevant time period
α^S	The decremental wane in supply (-ve) during the relevant time period
RANDOM VARIABLES	
D_t	Demand realized for time period t
S_t	Supply realized for time period t
π_t	Profit for time period t
S_t^i	The specific value of supply for time period t
D_t^j	The specific value of demand for time period t
DECISION VARIABLES	
$S_t^{\sim}$	Supply target for time period t
$S^{\sim}$	Supply target per time period for the entire time series
ρ_t	Production realized for time period t
$\rho_t(S_{t-\Delta S}^i)$	Production realized for time period t as a function of supply realized
φ_t	The sale realized for time period t

Model: Stochastic Time-Series Optimization

$$max \sum_{t\in T}\sum_{i\in I} p_i^S * \sum_{j\in J} p_j^D * \pi_t$$

$$s.t. \pi_t = P * \varphi_t - \left(C_1 * S_{t-\Delta S}^i + C_2 * \rho_t(S_{t-\Delta S}^i) + C_3 * \varphi_t\right) \quad \forall\, t \in T, i \in I, j \in J$$

$$+ \left(G_1 * \left(S_{t-\Delta S}^i - \rho_t(S_{t-\Delta S}^i)\right) + G_2 * \left(\rho_t(S_{t-\Delta S}^i) - \varphi_t\right)\right) - B * \left(D_{t+\Delta D}^j - \varphi_t\right)$$

$$\varphi_t \leq \rho_t(S_{t-\Delta S}^i) \quad \forall\, t \in T, i \in I$$

$$\varphi_t \leq D_{t+\Delta D}^j \quad \forall\, t \in T, i \in I$$

$$\rho_t(S_{t-\Delta S}^i) \leq S_{t-\Delta S}^i \quad \forall\, t \in T, i \in I$$

$$D_t^j = \mathrm{D} * \delta_j^D * (1 + Y_t * \alpha^D) \quad \forall\, t \in T, j \in J$$

$$S_t^i = S_t^{\sim} * \delta_i^S * (1 + Y_t * \alpha^S) \text{ OR} \quad \forall\, t \in T, i \in I$$

$$S_t^i = S^{\sim} * \delta_i^S * (1 + Y_t * \alpha^S)$$

$$Y_t \in \{0,1\} \quad \forall\, t \in T$$

$$\sum_{i\in I} p_i^S = 1$$

$$\sum_{j\in J} p_j^D = 1$$

The objective function intends to maximize the cumulative profit across the time series weighted by the probability of supply and demand for all scenarios. The profit function for each time period incorporates the supply and production realized for each scenario. The constraints can be explained as follows: we cannot sell more than we can produce, which depends on the probabilistic supply received; we cannot sell more than the probabilistic demand; we cannot produce more than we receive as the probabilistic supply. Finally, the sum of all probabilities must be 1 for both supply and demand.

Results

We solve the model for different scenarios of time series with or without festival days (denoted as 1 for festival day and 0 for the regular day). For a sample time series with or without a festival day, we note the impact of having supply targets set at time-period level vs entire time-series level.

Table 3: Supply Target Recommendations

Days	Supply Targets (A)	Total/Avg Profit	Supply Target (B)	Total/Avg Profit
-	105.263	3758.185	105.263	3758.185
-	105.263	536.884	105.263	536.884
0	105.263		105.263	
0	105.263		105.263	
0	105.263		105.263	
0	105.263		105.263	
0	105.263		105.263	
0	-		-	
0	-		-	
-	105.263	3865.562	110.000	3358.814
-	105.263	552.223	110.000	479.831
0	126.316		110.000	
0	105.263		110.000	
1	116.959		110.000	
0	105.263		110.000	
0	105.263		110.000	
0	-		-	
0	-		-	

We validate the solution via simulation using multiple scenarios of demand and supply for the same time series. We evaluated the resulting decision variables regarding supplier targets and production plans over multiple iterations. The simulation outcome converges to expected value within a tolerance of 0.5%.

Academic Contribution

Our extensive review of literature in this field indicates that most of the research has focused on these dimensions in isolation. Integrating two or more dimensions in the same research is mostly sparse. It is evident that there is a need to extend the existing research boundary to cover the unique problem of intermittent demand surges leading to demand-supply mismatches for perishables with supply variability, in the context of the dairy industry. Our exhaustive review of papers covering all three dimensions of this supply chain problem brings up no specific research done to address this challenge.

In effect, this thesis makes the following contributions to literature in the area of perishables inventory management:

- Development of a mathematical model that captures the impact of demand-supply mismatch within a time series of interest to the operations team.
- Formulation of a three-stage stochastic programming approach to address the decisions at sourcing, production and dispatch stages.
- Usage of open source alternatives such as Python and Google OR-Tools to build a working code that

solves a wide variety of scenarios of practical relevance within acceptable computational times.

Conclusions

In this research, we address the unique problem faced by companies dealing with perishable products when they face an intermittent demand-supply mismatch on account of festivals or similar occasions. At the same time, we face demand and supply uncertainty, which increases the complexity of the problem. We look at the dairy industry in eastern India using the company Milk Mantra as a case study to understand the problem, its causes, and its impact. Interviews with Procurement, Production and Sales teams provide adequate insight into the scale and complexity of the problem. Interactions with the suppliers provide a better understanding of the supply variability that comes on account of small farmers constituting the bulk of the supplier base. The need to set aside additional production for self-consumption during the festival times leads to a reduction in supply. Around the same time, demand from the consumer side goes up.

Stochastic optimization over the time series is used to arrive at the recommended supply targets. In order to make use of the power of linear programming, the stochastic distribution for demand and supply is broken down into a discrete empirical distribution. When presented with a time series of interest where specific festival days are marked, the solution presents the supply targets to be communicated to upstream players. We also notice that ensuring constant supply target for the entire time series alters the supply targets dramatically and has a profound impact on the overall profitability. On the other hand, enforcing the policy of fixed production targets for a time period does not impact the supply targets, but it does reduce the overall profitability. These results indicate the premium that we can expect to place on flexible suppliers and flexible production options.

This thesis creates a framework for businesses and researchers dealing with similar situations and provides a starting point for implementing stochastic optimization via linear programming as a tool for dealing with uncertainty around perishables.