ENDOGENOUS DEMAND IN SUPPLY CHAIN NETWORK DESIGN

BEN LILLIJORD AND ALAN LI

MIT SCM CAPSTONE PROJECT

MIT Center for
Transportation & Logistics

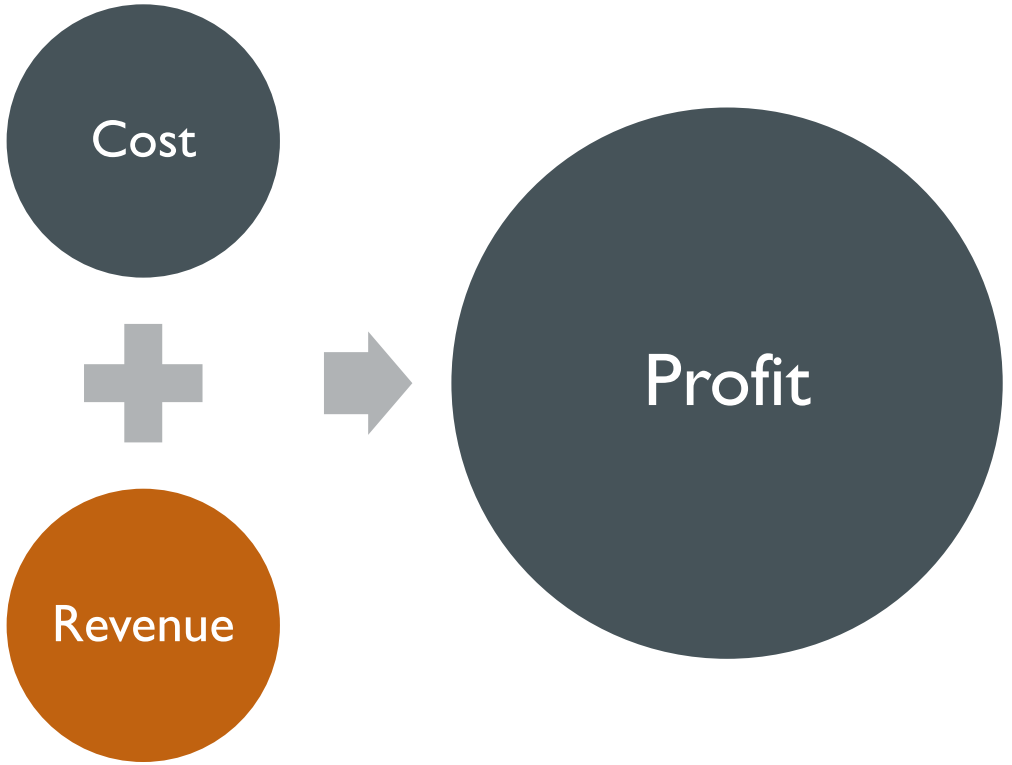
AGENDA

Introduction	• Motivation • Cost minimization vs profit maximization
Methodology	• Model outline • Endogenous demand incarnations
Analysis	• Sensitivity analysis • Results
Conclusion	• Summary • Future Research

INTRODUCTION
MOTIVATION – LOOKING BEYOND COSTS

- What happens when a new facility opens?
  1. Costs incurred
  2. Costs saved

- Any change in revenue?
  1. Better service
  2. Brand recognition



INTRODUCTION
MOTIVATION – PROFIT MAXIMIZATION

Cost Minimization

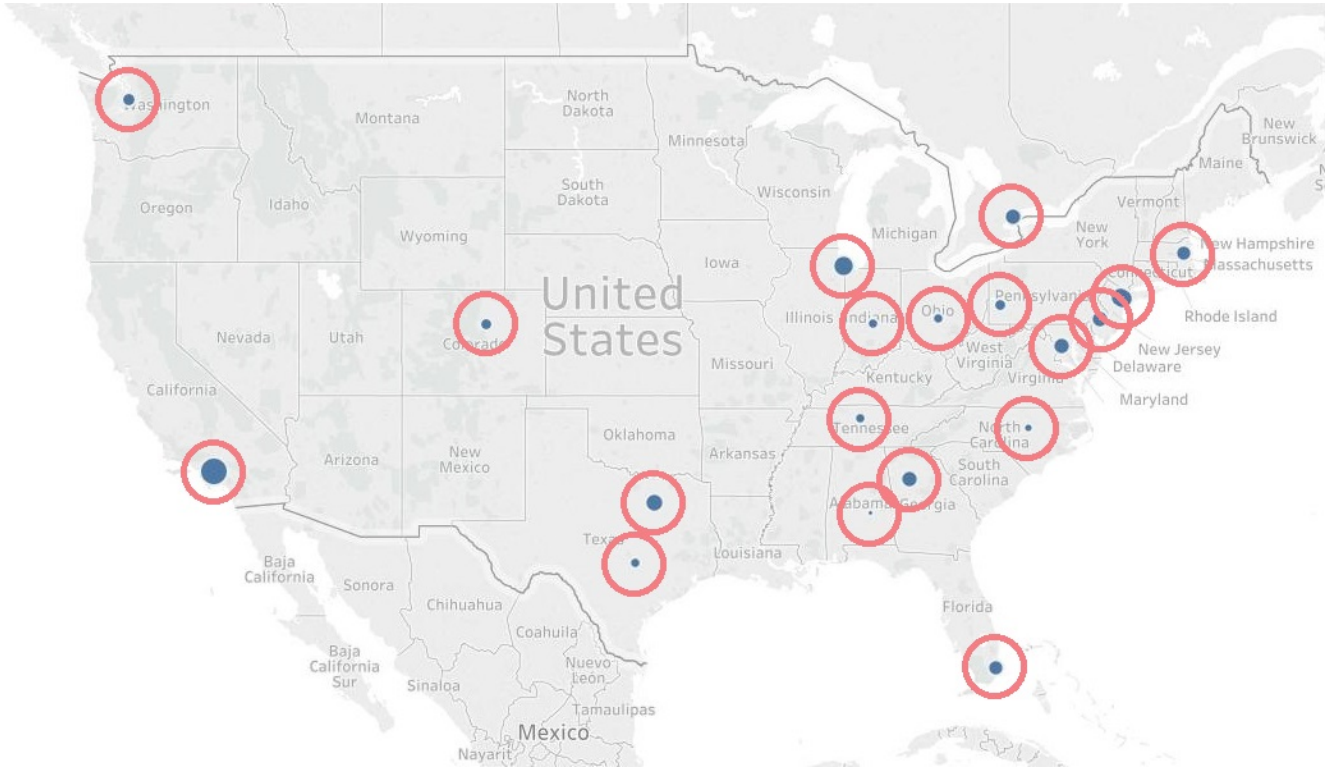
- Finds low-cost network to meet expected demand
- Considers only costs
- Revenue generation is incidental

Profit Maximization

- Demand changes depending on network design (Endogenous Demand)
- Considers costs and revenues
- Revenue generation is optimized

ANALYSIS
STEP FUNCTION ENDOGENOUS

- Simplest endogenous model
- Represents same-day shipping
- Demand increases by a set amount for customers near the facility



METHODOLOGY
OBJECTIVE FUNCTIONS & CONSTRAINTS

Objective Functions

Cost minimization objective function:

$$\sum_{i=1} f_i Y_i + \sum_{i=1}\sum_{j=1} c_{ij}(d_{ij}) x_{ij}.$$

Facility Fixed Cost, Delivery Cost

Profit maximization objective function:

$$\sum_{i=1}\sum_{j=1} [r_{ij} - c_{ij}(d_{ij})] x_{ij} - \sum_{i=1} f_i Y_i.$$

Revenue per unit

Common Constraints

Capacity constraint

$$\sum_j x_{ij} \le k_i, \forall i \in I.$$

Linking constraint

$$\sum_j x_{ij} \le M Y_i, \forall i \in I\,.$$

Binary constraint for facilities:

$$Y_i = \{0, 1\}, \forall i \in I.$$

All flow of product must be non-negative:

$$x_{ij} \ge 0, \forall i \in I, \forall j \in J.$$

Demand Constraints

Traditional demand:

$$\sum_i x_{ij} \approx D_j, \forall j \in J.$$

Endogenous demand

$$\sum_i x_{ij} \approx D_j + \sum_i ED_{ij}(d_{ij}) \forall j \in J.$$

METHODOLOGY
ENDOGENOUS DEMAND EFFECT FUNCTION & COEFFICIENT

The general form of the endogenous demand effect function: $\sum_i ED_{ij}(d_{ij}) = \sum_i \frac{\sigma_j Y_i E_{ij}(d_{ij}) x_{ij}}{D_j}, \forall j \in J.$

Quadratic Form

The endogenous demand constraint becomes: $\sum_i \left[1 - \frac{\sigma_j Y_i E_{ij}(d_{ij})}{D_j}\right] x_{ij} \approx D_j, \forall j \in J.$

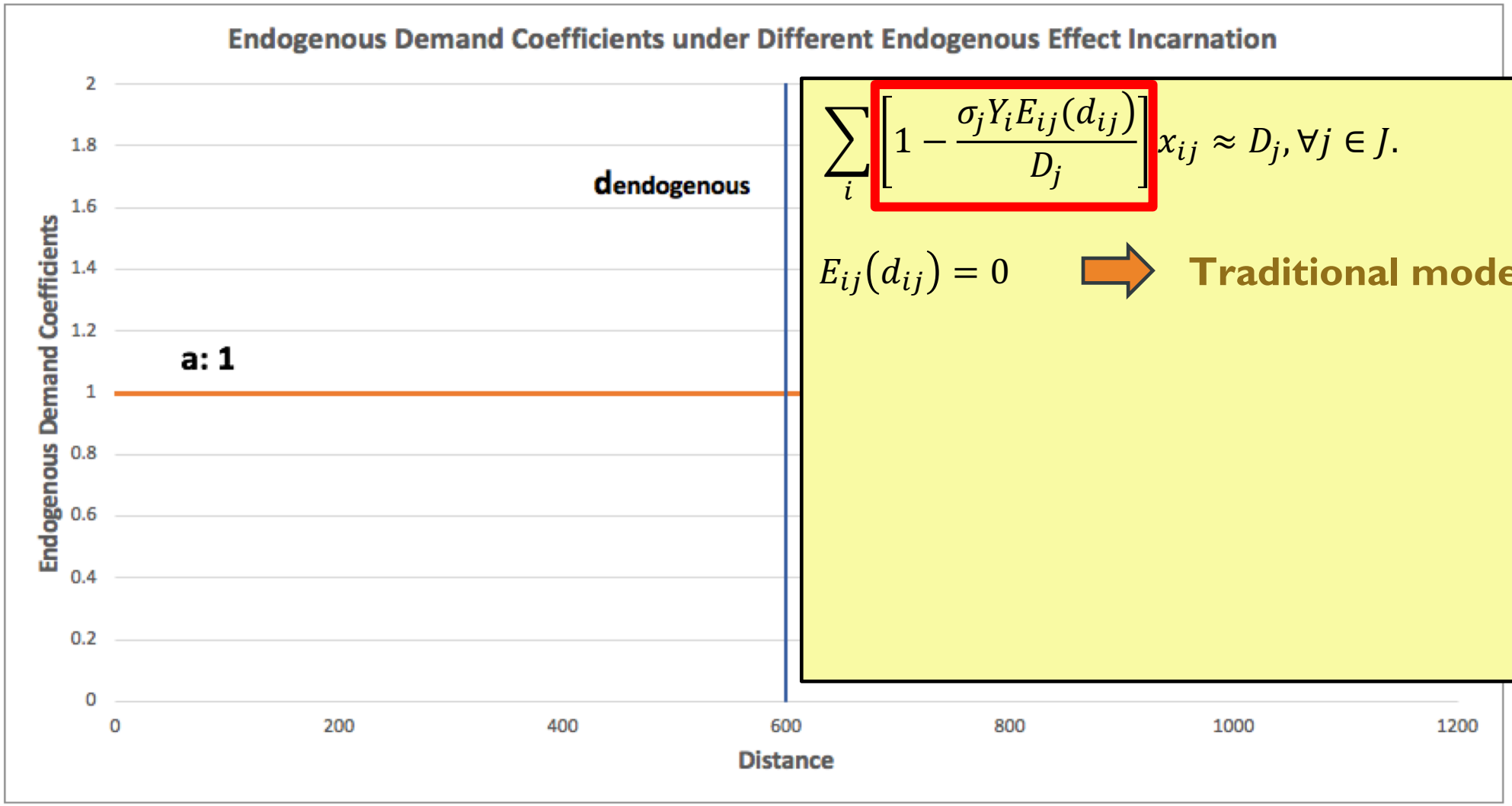
Endogenous Demand Coefficient

Where:

σ_j is the variation of the demand from customer locations j, for $j \in J$.

$E_{ij}(d_{ij})$ describes the endogenous effect incarnation.

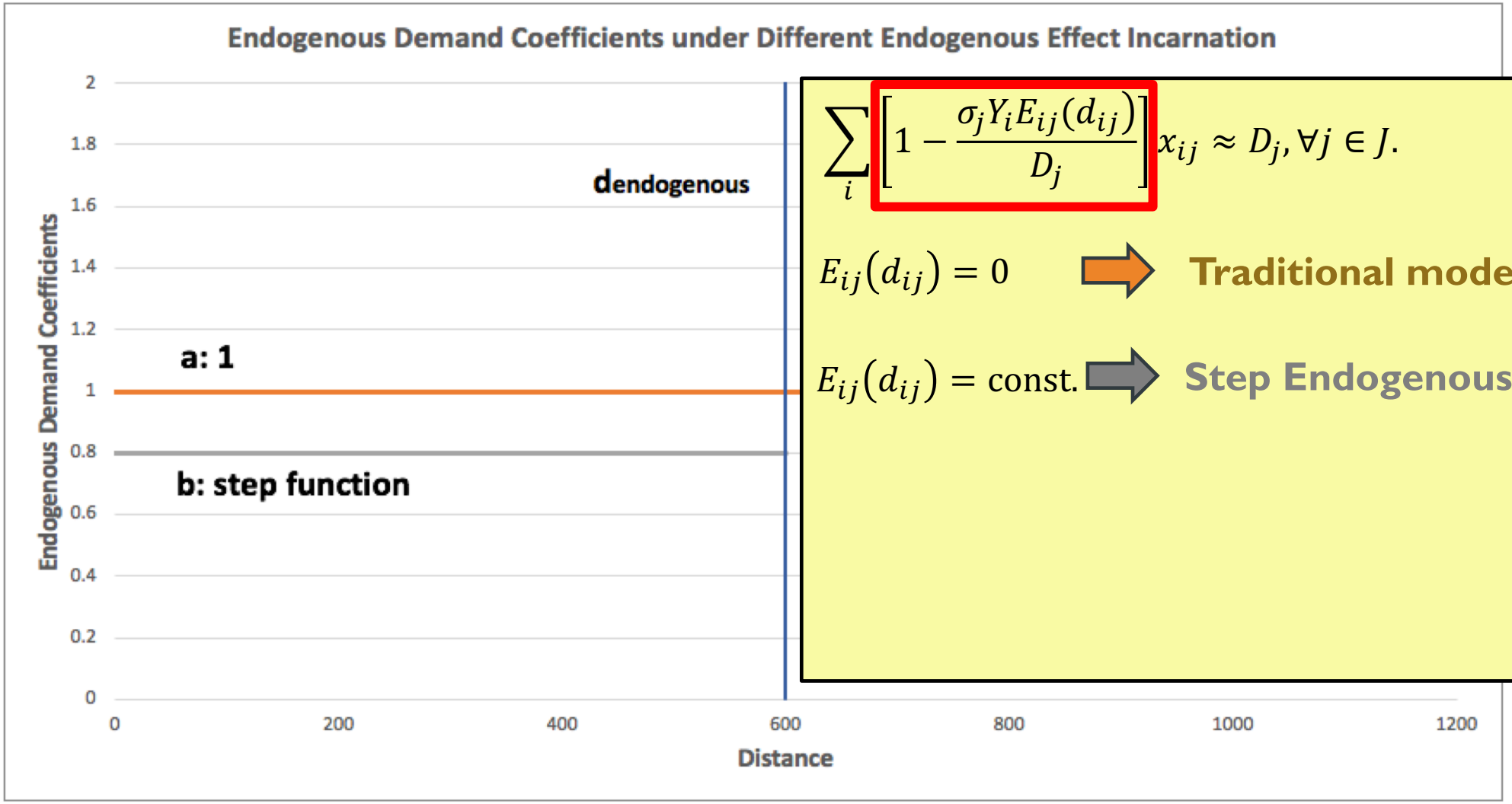
METHODOLOGY
VARIOUS ENDOGENOUS DEMAND COEFFICIENT



$$\sum_i \left[1 - \frac{\sigma_j Y_i E_{ij}(d_{ij})}{D_j}\right] x_{ij} \approx D_j, \forall j \in J.$$

$E_{ij}(d_{ij}) = 0$ ⇒ **Traditional model (a)**

METHODOLOGY
VARIOUS ENDOGENOUS DEMAND COEFFICIENT

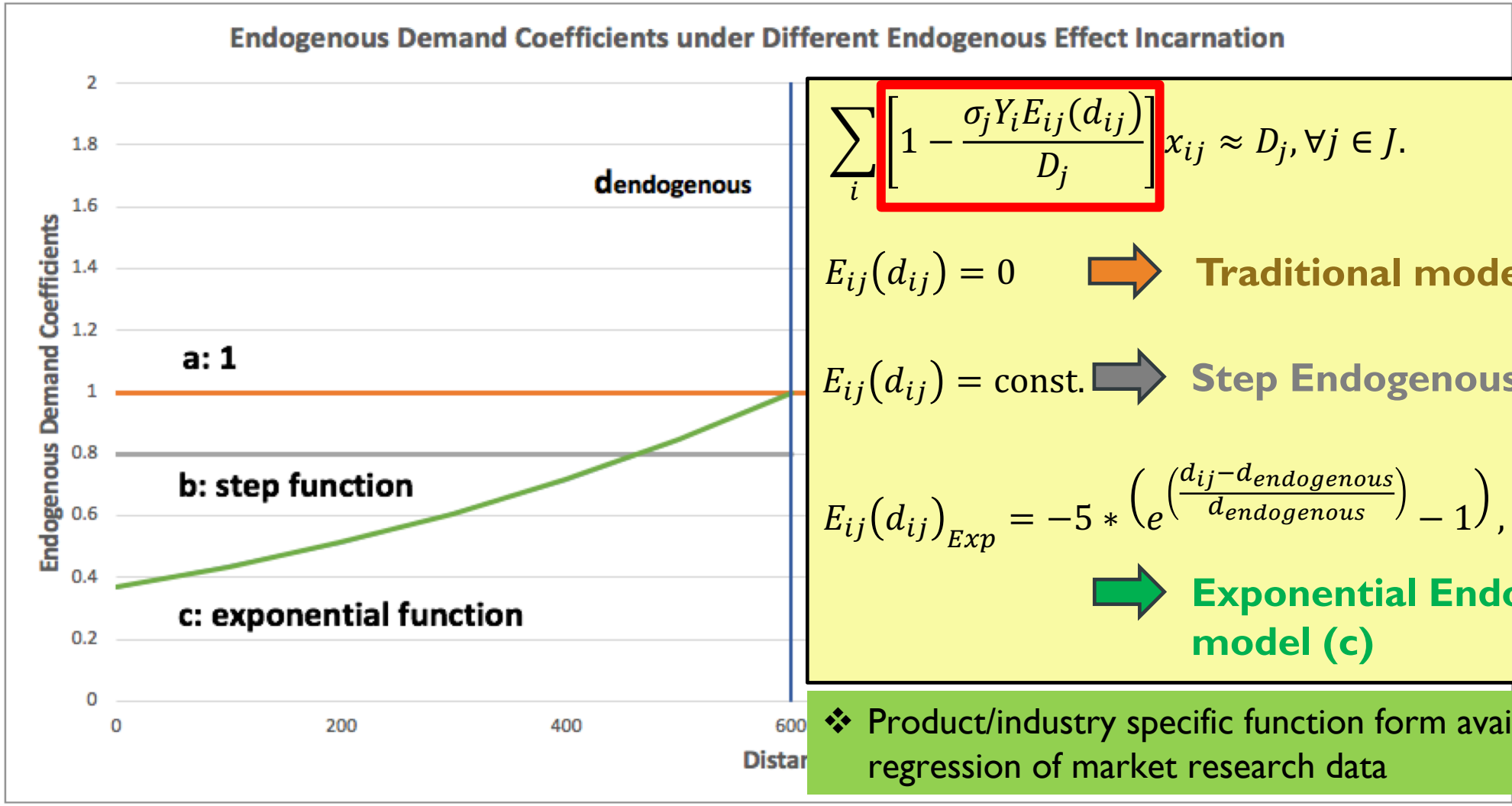


$$\sum_i \left[1 - \frac{\sigma_j Y_i E_{ij}(d_{ij})}{D_j}\right] x_{ij} \approx D_j, \forall j \in J.$$

$E_{ij}(d_{ij}) = 0$ ➡ **Traditional model (a)**

$E_{ij}(d_{ij}) = \text{const.}$ ➡ **Step Endogenous model (b)**

METHODOLOGY
VARIOUS ENDOGENOUS DEMAND COEFFICIENT



$$\sum_i \left[1 - \frac{\sigma_j Y_i E_{ij}(d_{ij})}{D_j}\right] x_{ij} \approx D_j, \forall j \in J.$$

$E_{ij}(d_{ij}) = 0$ ➡ **Traditional model (a)**

$E_{ij}(d_{ij}) = \text{const.}$ ➡ **Step Endogenous model (b)**

$$E_{ij}(d_{ij})_{Exp} = -5 * \left(e^{\left(\frac{d_{ij} - d_{endogenous}}{d_{endogenous}}\right)} - 1\right),$$

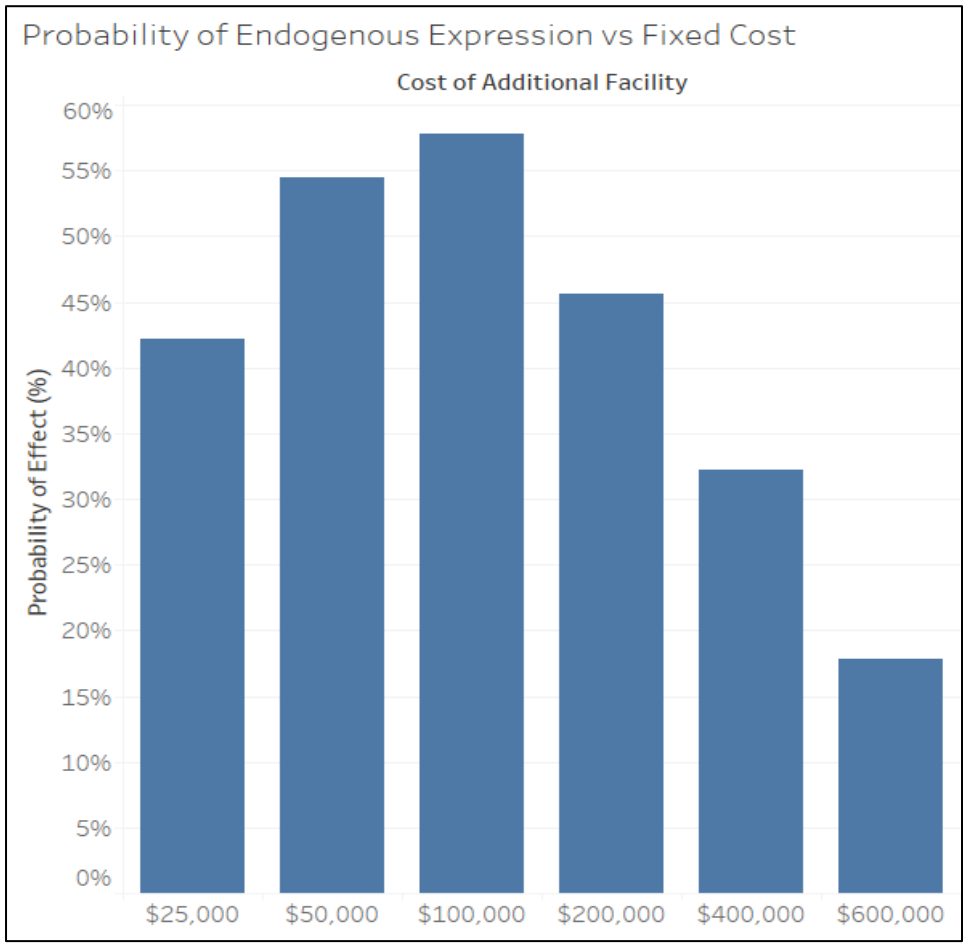
➡ **Exponential Endogenous model (c)**

❖ Product/industry specific function form available from regression of market research data

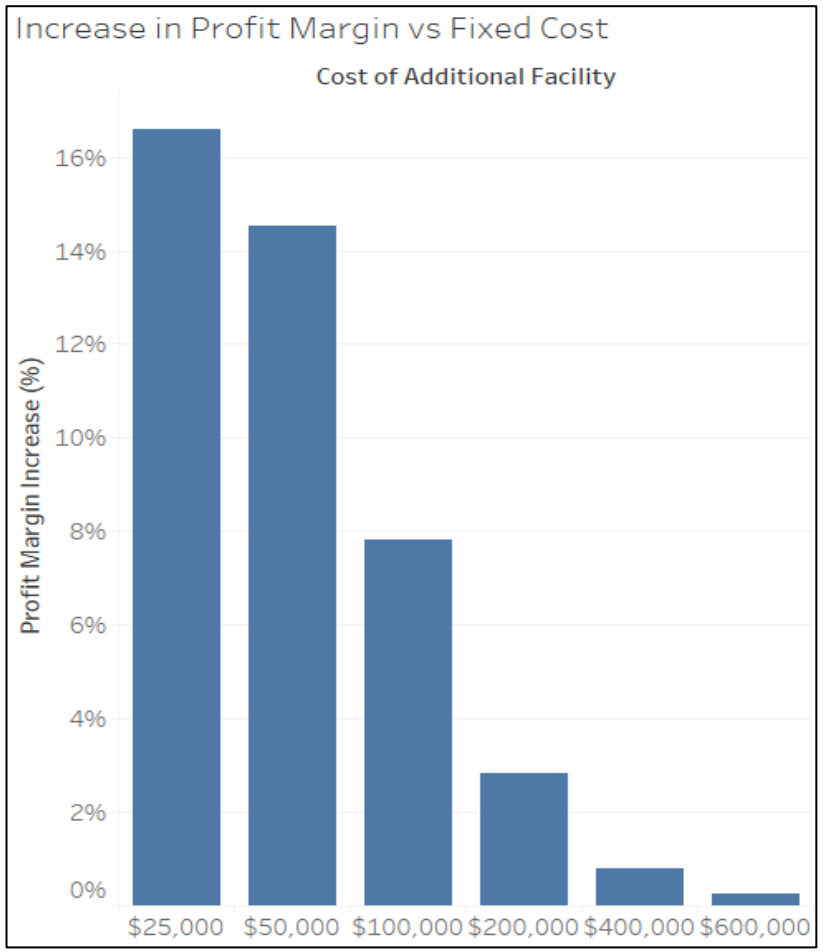
ANALYSIS
SENSITIVITY ANALYSIS

- The analysis is performed in Python with IBM CPLEX Optimizer. Each combination of metrics was tested in a total of 40,320 runs and 1,260 optimal solutions in the sensitivity analysis alone.
- Assumptions
  - Unconstrained capacity
  - Step Endogenous Incarnation
  - Focus on network design corresponding fixed cost and delivery (linehaul) cost
  - Direct distance by using Haversine formula with longitude and latitude of each city
  - Demand in each region is simplified to be concentrated and located in the city centroid
- Combination of matrices
  - Fixed cost of an additional facility (values from $25,000 to $600,000).
  - Variable per mile cost of shipping goods (values from $0.005 to $0.15).
  - Total demand at each demand center (values from 2,000 to 20,000).
  - Variable demand from endogenous effects (values from 300 to 1,500).

ANALYSIS
ENDOGENOUS EXPRESSION VS FIXED COST OF FACILITY

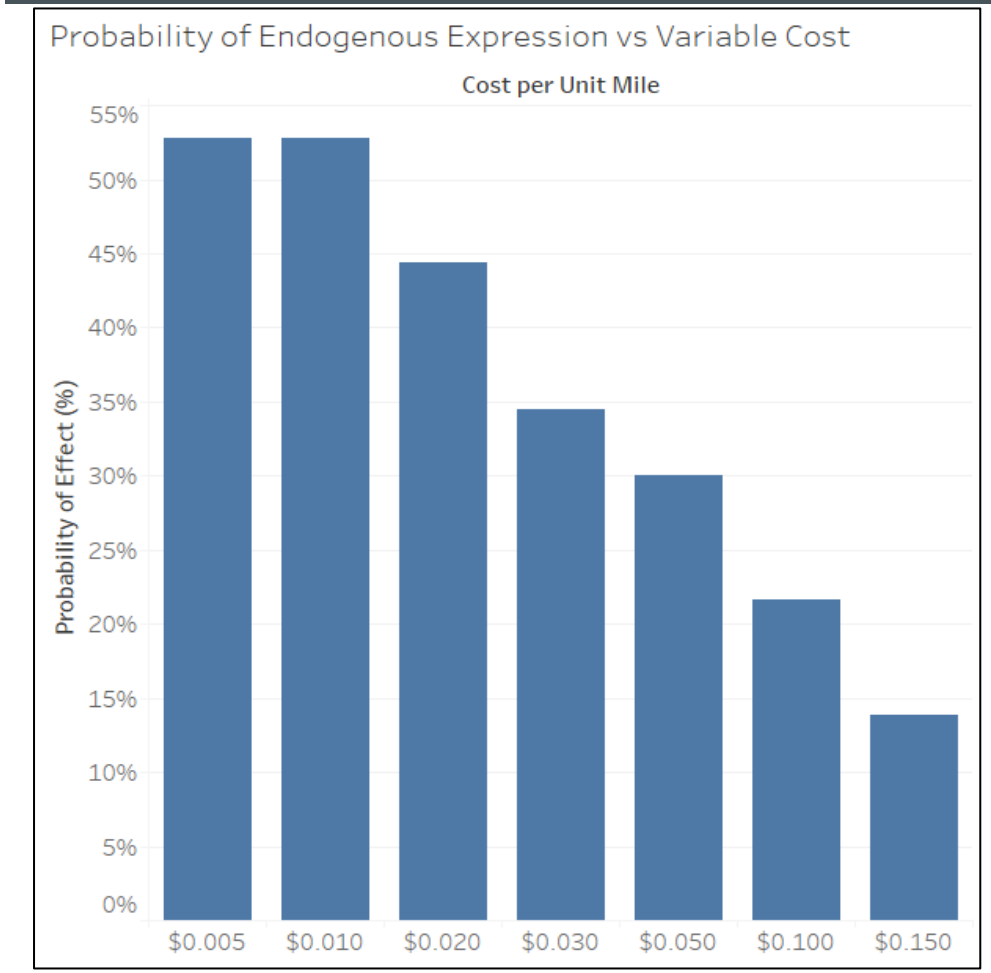


Probability

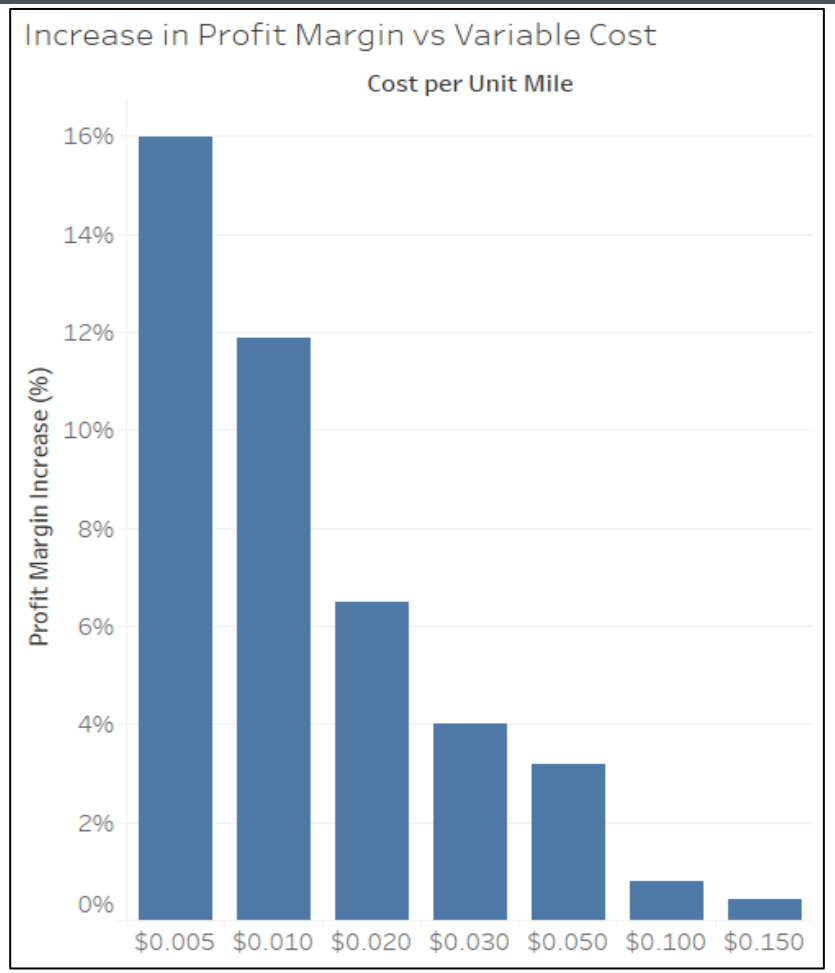


Profit Margin Improvement

ANALYSIS
ENDOGENOUS EXPRESSION VS COST PER UNIT MILE

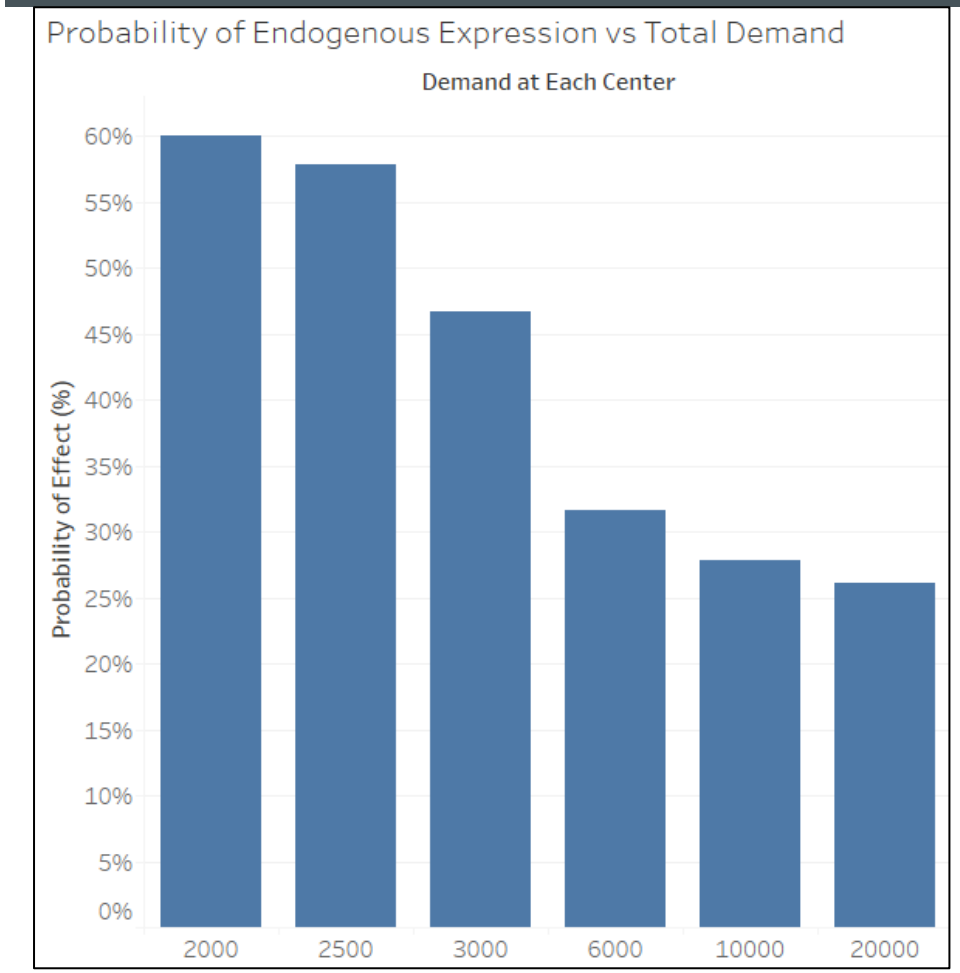


Probability

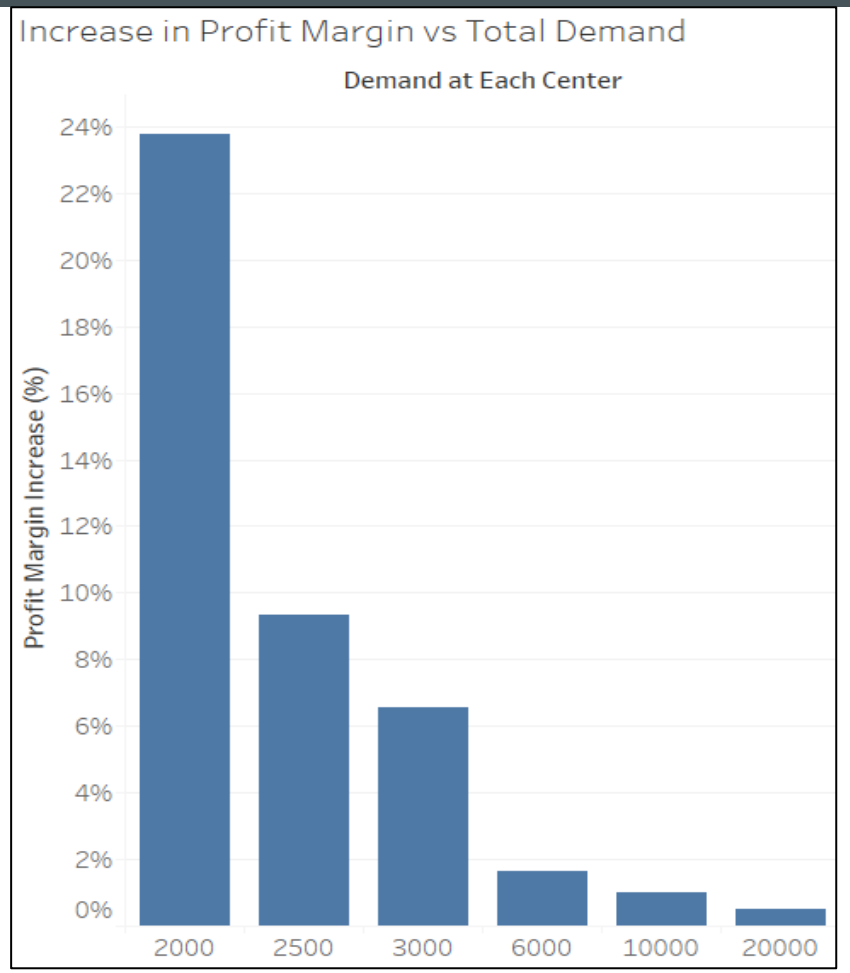


Profit Margin Improvement

ANALYSIS
ENDOGENOUS EXPRESSION VS TOTAL DEMAND

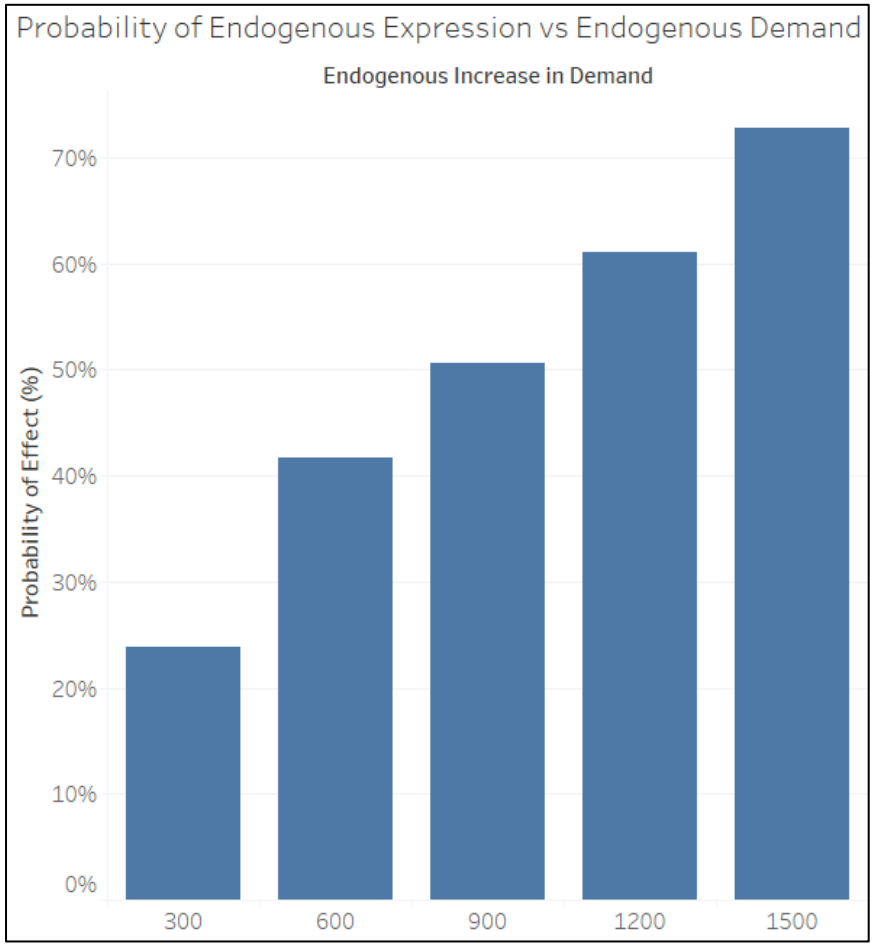


Probability

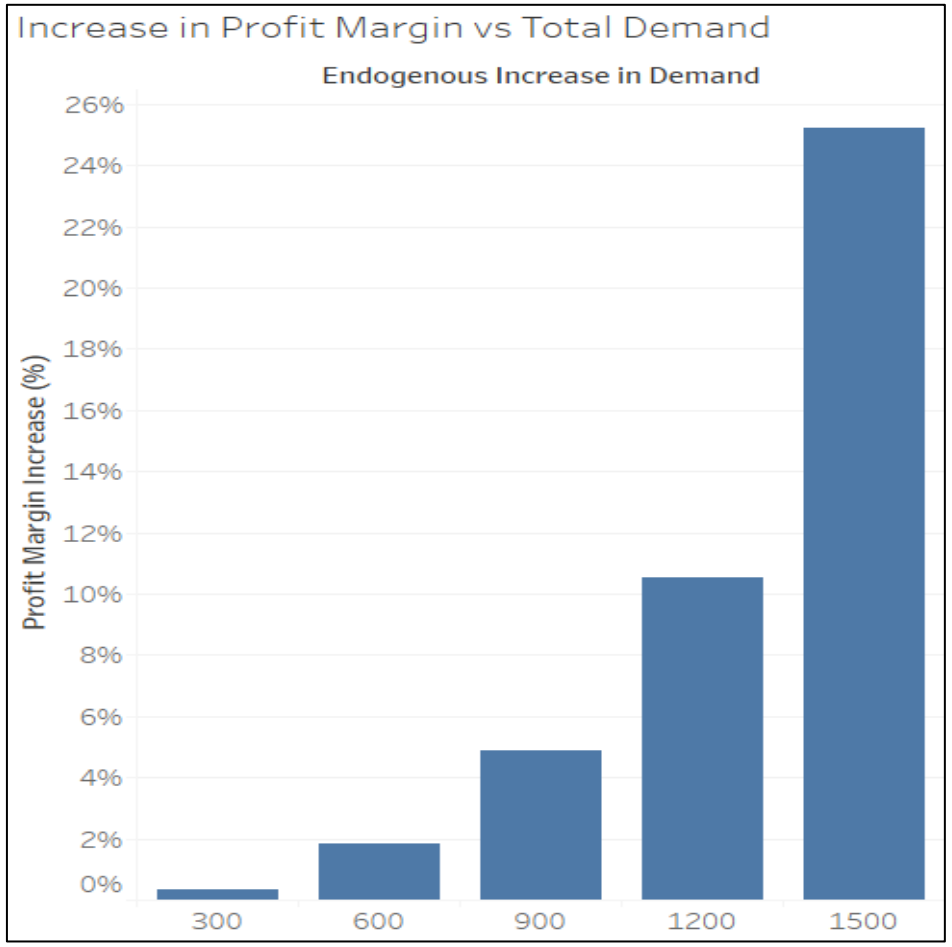


Profit Margin Improvement

ANALYSIS
ENDOGENOUS EXPRESSION VS VARIABLE DEMAND



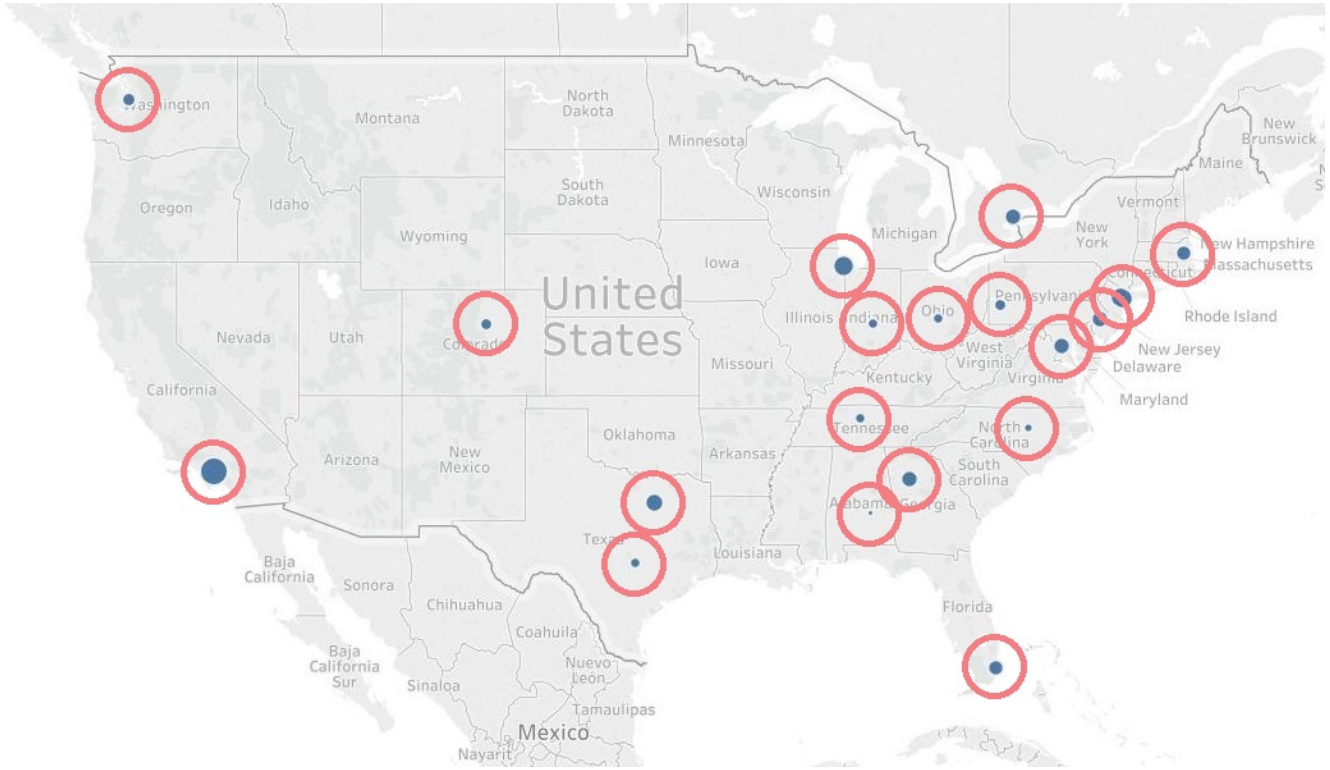
Probability



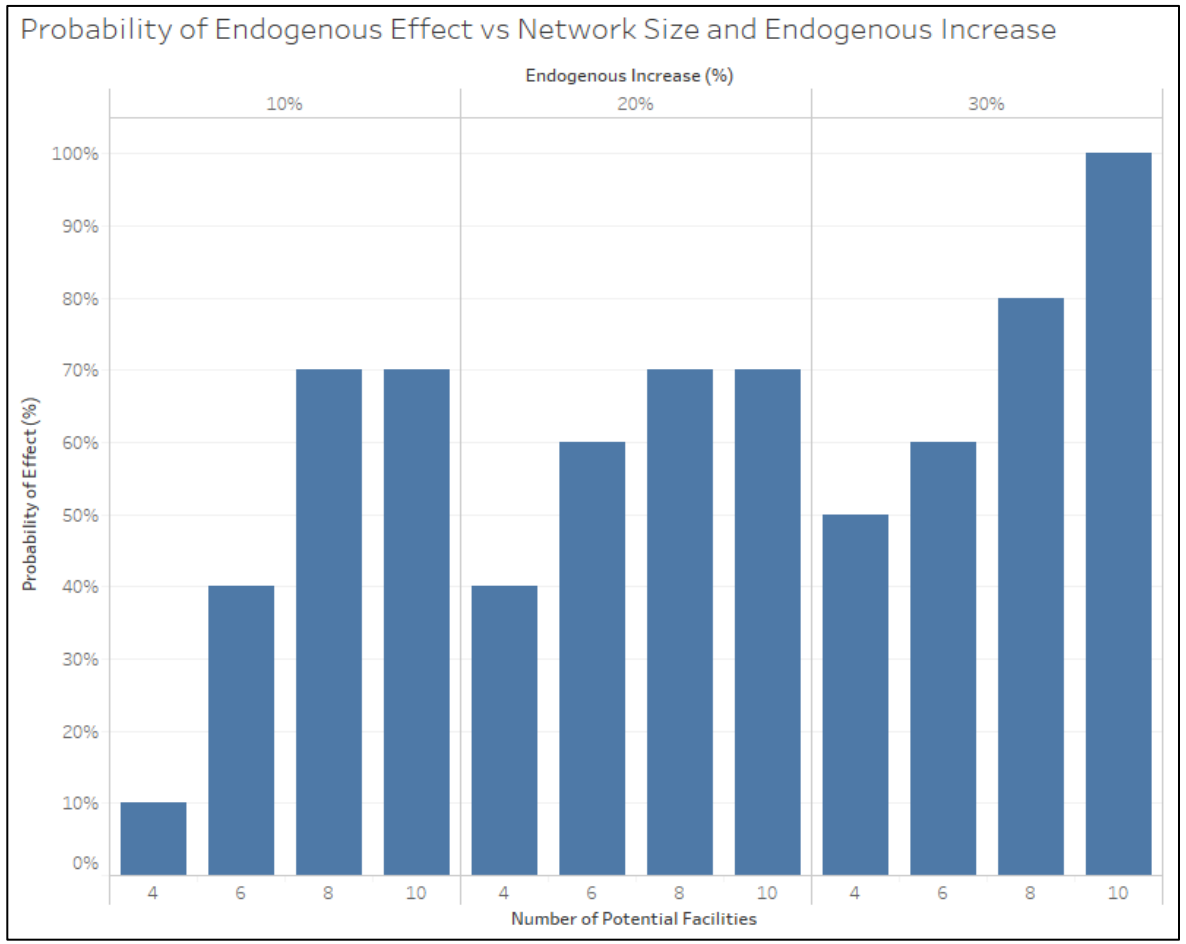
Profit Margin Improvement

ANALYSIS
STEP FUNCTION ENDOGENOUS

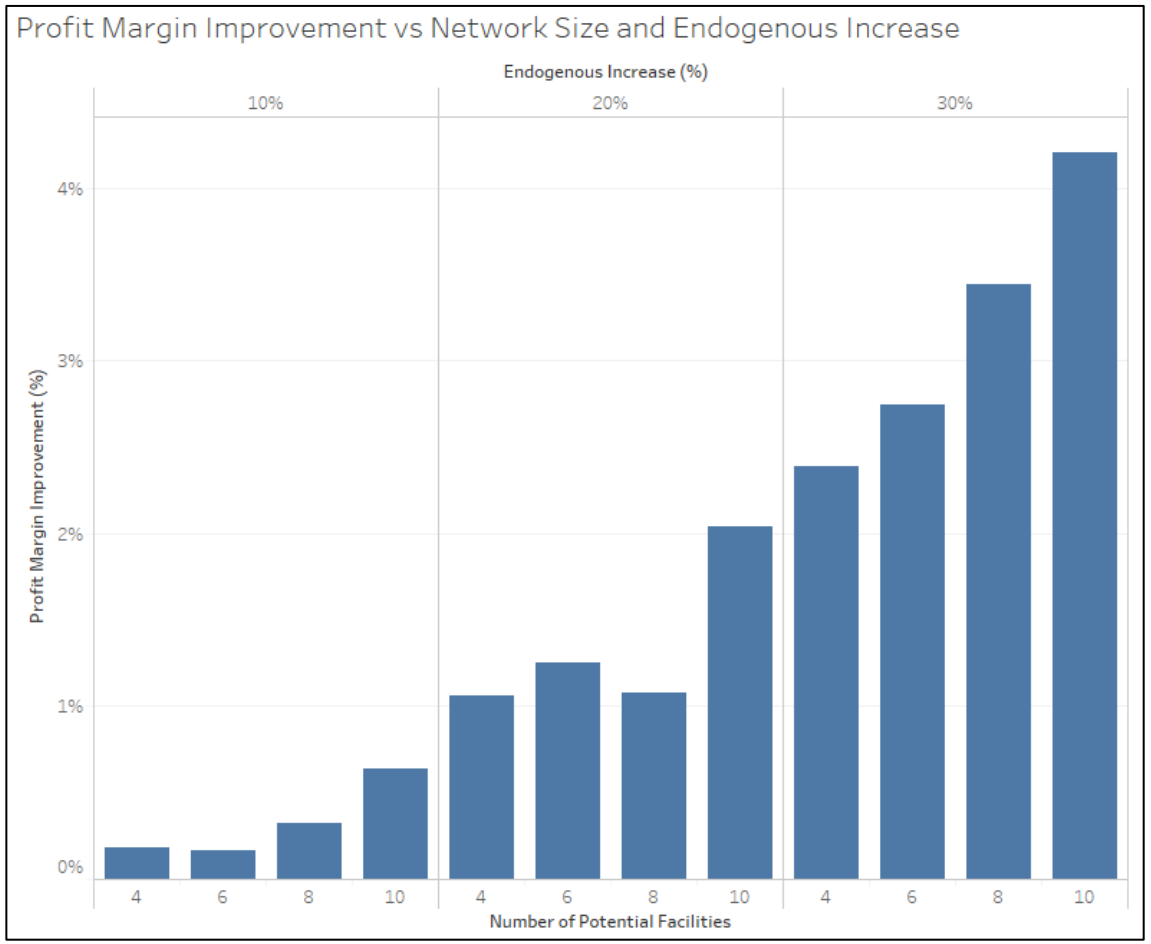
- Profit increases as high as 10.2%
- Under favourable conditions, average profit increase of 3.2%
- More profitable network found in nearly half of the simulations at only 10% endogenous effect
- Complex networks tend to see greater benefits



ANALYSIS
ENDOGENOUS EXPRESSION – STEP FUNCTION



Probability

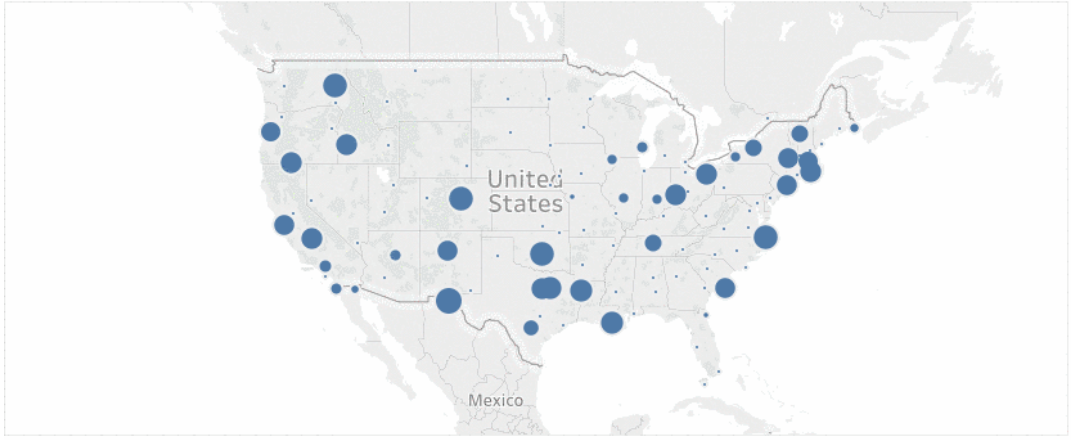


Profit Margin Improvement

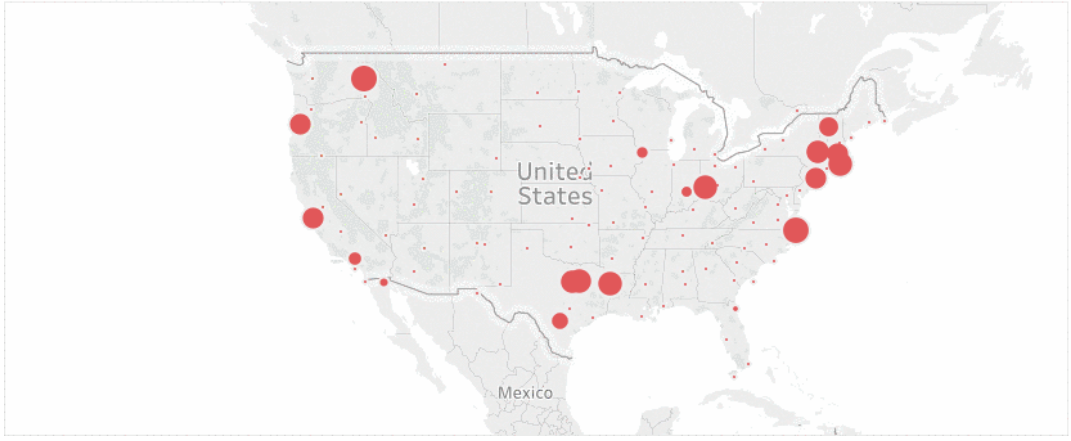
ANALYSIS
NETWORK EXPANSION PROBLEM

- Common problem: decision to add one or more facilities
- Average Return On Assets increased by 10% with profit maximizing model
- Promotes faster network expansion
- The profit maximization model often chose locations that were never chosen by the cost minimization model

Profit Maximizing



Cost Minimizing



CONCLUSION
SUMMARY OF RESULTS

- Three significant findings:

1. Even small endogenous effects result in network changes
2. The profit maximization model was more valuable in complex networks
3. Profit maximization model tends to place facilities closer to demand centers

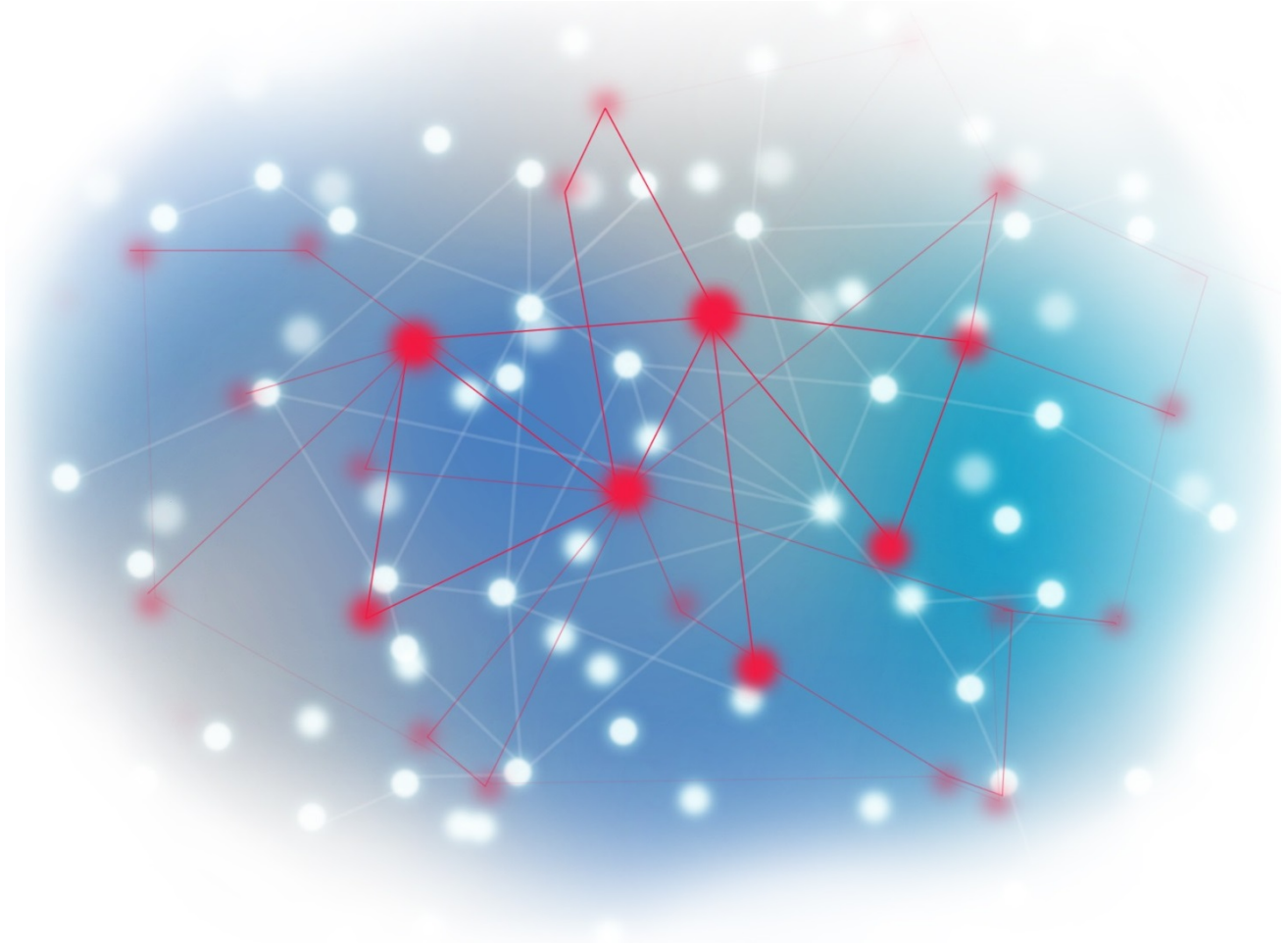
CONCLUSION
FUTURE RESEARCH

- Applications
  - Perform demand analysis for specific industry to estimate potential for endogenous effect and model incarnation
- Capacitated facility study
  - Use the uncapacitated model in this thesis as a basis, future research can work on capacitated problem
- Distance for route between each pair of facility and client
  - Change the direct distance by using Haversine formula with longitude and latitude to real route with google map API

THANK YOU
QUESTIONS?

Ben Lillijord

Bachelor of Science
Mechanical Engineering
University of Alberta

Alan Li

Master of Science
Applied Mechanics
National Taiwan University