

# Endogenous Demand in Supply Chain Network Design

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**Summary:** Typical supply chain networks are optimized with a simple goal: find the lowest cost system that can meet a given expected demand. Formulating the problem in this way leads to a relatively easy and intuitive network optimization problem, but it does not consider the network's impact on demand in the system as a whole. This research sought to investigate ways in which a network can impact demand in the system that surrounds it, and to formulate network optimization that considers this impact. Organizations can achieve higher profits by considering the demand driven by the network, known as "endogenous demand". The methodology presents a broad range of simulations that determine the magnitude of profits that can be achieved.



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## KEY INSIGHTS

1. Endogenous demand optimization is more likely to improve profitability as the system complexity increases.
2. Simulations found average profit increases up to 3.19% under favorable conditions.
3. Endogenous demand optimization will not always find new sources of profits, but profit improvements may emerge under a broad range of conditions.

## Introduction

Consider an online retailer with distribution centers across the United States. The retailer does not have a distribution center in Boston, and so takes about three days to deliver products to Boston customers. The retailer knows that if they open a distribution center there, they can gain customers by offering same-day shipping. In this example, the endogenous demand is any demand growth that the retailer experiences in Boston as a result of the same-day shipping offer. This

may seem to be an obvious factor, but it is not presently considered in supply chain optimization.

This research effort explores a range of simulated incarnations of endogenous demand in real-world situations. The purpose of this report is to compare the results of traditional supply chain optimization to endogenous supply chain optimization in a range of systems. To analyze the potential that endogenous demand presents, the research begins with a network optimization problem and two objective functions: minimize cost and maximize profit. This is equivalent to comparing traditional supply chain optimization (minimize cost) and endogenous supply chain optimization (maximize profit). With no endogenous demand in the system, both objective functions always result in the same solution. Various incarnations of endogenous demand are then added to the system, and the results of the traditional and endogenous optimization models are compared. These results demonstrate the increased profit that an organization can obtain by optimizing for endogenous demand.

## Methodology

The general model is a network optimization problem that sets out to supply a set of customers at end nodes from a set of facilities at start nodes. Costs are based on a fixed cost to build a facility at a given start node, and a variable cost for each item shipped from a start node to an end node multiplied by a distance factor. The objective functions are represented as:

Cost minimization objective function:

$$\sum_{i=1} f_i Y_i + \sum_{i=1} \sum_{j=1} c_{ij}(d_{ij}) x_{ij}.$$

Profit maximization objective function:

$$\sum_{i=1} \sum_{j=1} [r_{ij} - c_{ij}(d_{ij})] x_{ij} - \sum_{i=1} f_i Y_i.$$

Where:

- $i$  the set of facilities,  $i = 1, \dots, n$ .
- $j$  the set of customer locations,  $j = 1, \dots, m$ .
- $f_i$  the fixed cost of a facility at location  $i$ .
- $Y_i$  the binary variable for a facility to be open.
- $c_{ij}(d_{ij})$  the handling and delivery cost per unit.
- $x_{ij}$  the product delivery flow.
- $r_{ij}$  the revenue per product delivery.

Next, several endogenous demand incarnations proposed by this research are included. These are represented by the following constraints:

Endogenous demand constraint equation:

$$\sum_i x_{ij} \approx D_j + \sum_i ED_{ij}(d_{ij}), \forall j \in J.$$

General form endogenous demand effect function:

$$\sum_i ED_{ij}(d_{ij}) = \sum_i \frac{\sigma_j Y_i E_{ij}(d_{ij}) x_{ij}}{D_j}, \forall j \in J.$$

Where:

- $\sigma_j$  the variation of the customer demands.
- $E_{ij}(d_{ij})$  the endogenous effect incarnation.

Endogenous demand optimization problems present a nonlinear (quadratic) expression. In this research, however, an alternative approach was used which allowed the results to be optimized as a set of linear programs. The set included all combinations of facilities, with each input represented by a particular combination. A thorough investigation of each parameter set with all possible facility combinations was then performed. This resulted in a sound methodology to investigate endogenous demand.

## Sensitivity Analysis

The model analysis began with a sensitivity analysis to determine the set of parameters in which endogenous optimization was likely to result in a more profitable network. The results from the sensitivity analysis were used to create simulations and test endogenous demand optimization, as presented in the next section. Various networks were simulated to determine the probability that endogenous factors would affect the network (referred to as "endogenous expression") and the corresponding increase in profitability. Four metrics were analyzed across 40,320 simulations to find the cases in which endogenous demand has the greatest effect. These metrics were:

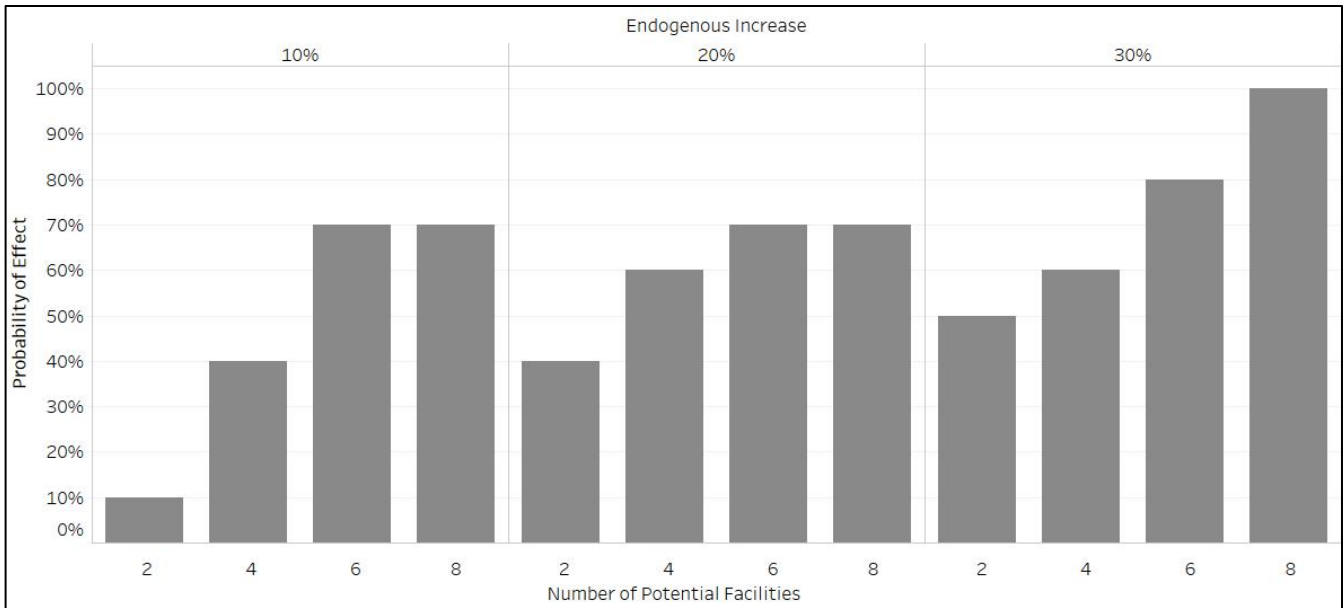
- Fixed cost of an additional facility
- Variable per mile cost of shipping goods
- Total demand from each customer
- Additional demand from endogenous

The simulations included all possible combinations of the metrics listed above, and used an underlying set of fixed facility and customer locations across the United States. Results indicated that endogenous expression is more likely under certain conditions, but hard cut-off values did not emerge. The conclusion that emerged was that profitability improvements are possible under a broad range of conditions. There were, however, conditions that were favorable toward endogenous expression. These included low shipping costs, mid-range facility costs, low total demand, and (as expected) high endogenous demand.

## Endogenous Expressions

The first endogenous demand function tested was a step increase in demand to any customer within a 100-mile radius around a shipping facility. The function represents an increase in demand associated with a business offering same-day shipping to its customers. To test the potential for endogenous demand optimization, a total of 40,800 simulations were run. These runs featured a range of potential facility locations and customer demand centers represented by coordinates of cities in the United States.

The network size was varied; the smallest of which featured 4 potential facilities and 8 customer demand centers served, and the largest featured 10 potential facilities and 20 customer demand centers served. Finally, the step change in demand was simulated using 10%, 20%, and 30% increases.



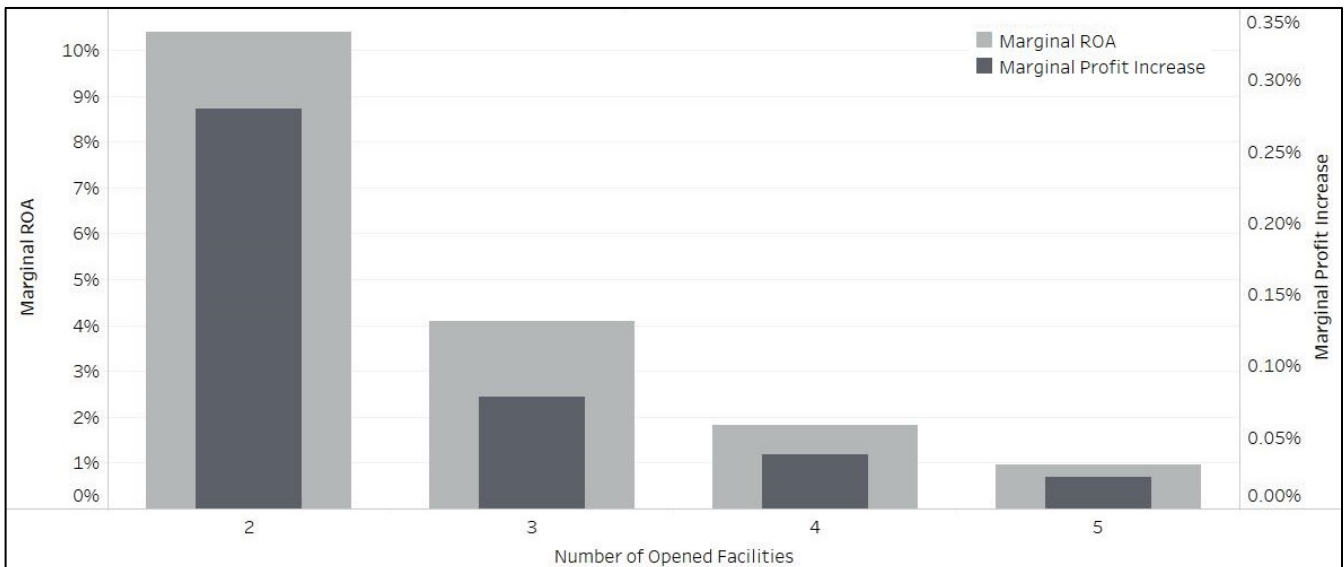
**Figure 1: Probability of Endogenous Expression under Step Function Endogenous Demand**

Figure 1 demonstrates that the probability of finding an endogenous expression increase as the complexity of the system increases. That is, systems of 10 facilities and 20 demand centers are more likely to benefit from endogenous demand optimization than those with 4 facilities and 8 demand centers.

In addition, probability of endogenous expression increases as endogenous effect increases. Under a 10% step increase in demand associated with endogenous effects, results indicate that the profit maximization model finds a more profitable network in 47.5% of networks. The average increase in profitability is 0.33%, and the increase in profitability ranges from 0% to 1.84%. The probability of

endogenous effect increases to 60.0% with a 20% step increase in demand. The average profitability increase is 1.39%, and the increase in profitability ranges from 0% to 4.69%. Under a 30% step increase in demand, probability of endogenous expression increases to 72.5%. Average increase in profitability is 3.19%, and the increases in profitability range from 0% to 10.31%.

An exponential endogenous expression was tested next, using an existing facility location network problem: A company that has one existing facility needs to determine which location(s) to expand its network in order to obtain the maximum potential profit. A total of 223,044 simulations were analyzed with 480 optimal solutions identified for analysis.



**Figure 2: Return on Assets and Profit Increase from Exponential Endogenous Demand**

Results from the exponential endogenous simulations are demonstrated in Figure 2. Significant profit improvements can be obtained by adding a single facility, but the benefits decrease as the system grows. This is likely because the locations that offer the greatest endogenous demand are purchased first, but additional facilities do not add more than the fixed cost of the facility. Interestingly, the simulations revealed that some facilities were chosen in all endogenous optimizations but never chosen in traditional cost minimization optimization.

A final endogenous demand expression that was simulated was the reverse endogenous demand that could emerge from low service levels. Customers are almost as sensitive to poor service as they are to price. As a result, they are likely to switch to another retailer if their expectations are not met. To reflect the customer expectation, the service level (and associated demand) is modelled to decrease as the distance increases beyond the effective distance. The endogenous exponential model more accurately represents the ROA relative to the baseline model in endogenous conditions, showing an ROA decrease from 96.2% to 91.6% as demonstrated in Figure 3.

### Conclusion

This report investigates the ways in which a network can impact demand in the system that surrounds it, and formulates network optimization models that considers these impacts. Several incarnations of these endogenous demand factors were simulated based on discussions with industry professionals in an attempt to model real-world situations.

The research uncovered two surprising findings. First, that even small levels of endogenous demand often result in a change in network optimization to a more profitable structure (10% endogenous effect yielded a change in 47.5% of simulations). Second, that more complex networks tend to attain greater benefits from profit maximization models than simple networks (an average profit increase of 3.19% in the most complex simulations). Combined, these findings suggest that companies with complex networks are likely to benefit from including endogenous demand factors, even if their particular incarnation does not appear to have a large magnitude.

Future researchers are encouraged to obtain company data and to create incarnations of endogenous demand directly. A more direct representation may provide insights beyond those found in the simulations presented in this research.

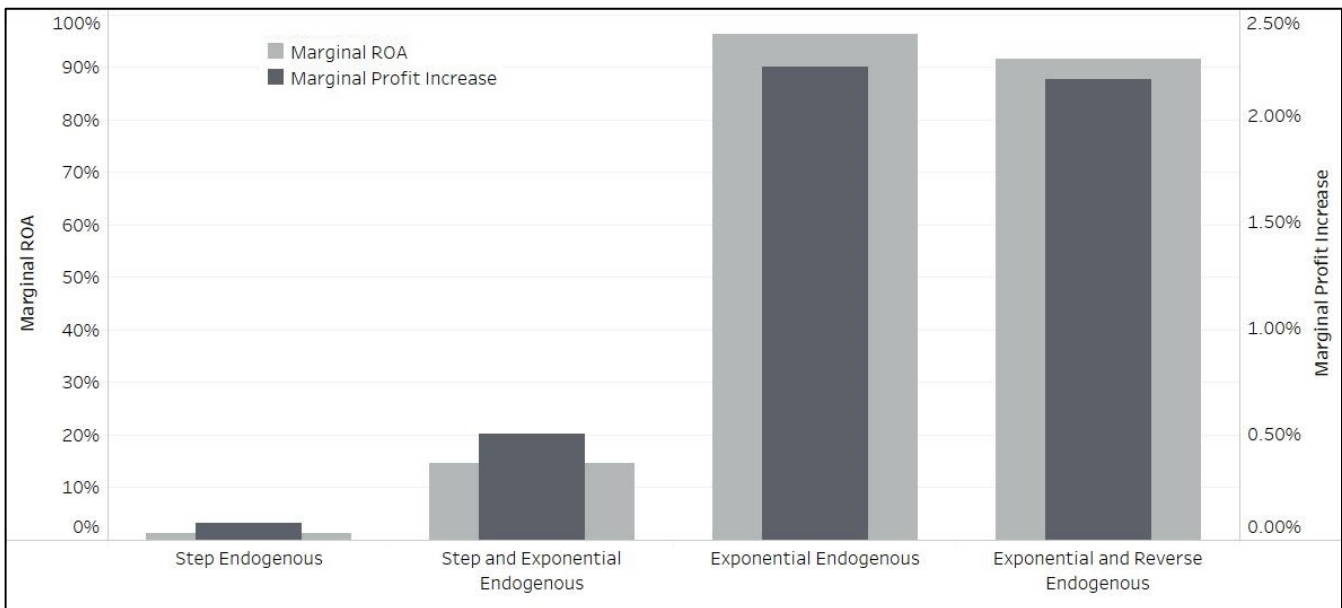


Figure 3: Return on Assets and Profit Increase under Reverse Endogenous Demand